Physics 407-09
Assignment 6

1) Show that the vectors K_1^A and K_2^A with components

$$K_1^t = K_1^r = 0 \tag{1}$$
$$K_1^\theta = \cos(\phi) \tag{2}$$
$$K_1^\phi = -\sin(\phi)\cot(\theta) \tag{3}$$

and

$$K_2^t = K_2^r = 0 \tag{4}$$
$$K_2^\theta = \sin(\phi) \tag{5}$$
$$K_2^\phi = \cos(\phi)\cot(\theta) \tag{6}$$

are Killing vectors of the two dimensional metric

$$ds^2 = d\theta^2 + sin(theta)^2 d\phi^2$$

Show that the Lie derivative of K_1^A by K_2^A is the third rotational Killing vector whose ϕ component is 1 and others are zero.

2.a) Find the radial geodesic equations for light emitted from r=0 at $t = t_1$ and absorbed at $r = R$ at time $t = t2$ in the standard t, r, θ, ϕ coordinates for the homogeneous and isotropic cosmological spacetimes.

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2(d\theta^2 + sin(\theta)^2 d\phi^2))$$

Assume $\theta = \pi/2$. Given a divergence $\delta\phi$ in two light rays from the source, what is the spatial distance between them at $t = t_2$ and $r = R$. Assume that the light is emitted uniformly from the star at $r = 0,\ t = t_1$, how does the intensity of the light drop off as a function of R? (If N photons per second are emitted uniformly in direction from the star at time t_1 how many of them will cross a unit area in unit time at $t = t_2$ and $r = R$?)

b) Show that the curve r=0 is a timelike geodesic.

3.Consider a flat, dust filled universe, for which $a(t) = a_0 t^{2/3}$. Write the metric in terms of the area coordinate R defined so that the angular part of the metric is $R^2(d\theta^2 + sin(\theta)^2 d\phi^2)$, and t, the normal cosmological time.