Physics 407-09
Assignment 5

1) Show that for the metric

$$ds^2 = (1 - \frac{2M}{r})(-dt^2 + dr^2 + r^2 d\theta^2 + r^2 sin^2(\theta) d\phi^2) \quad (1)$$

there is no deflection of light.

(solve the geodesic equations, where you can again assume that $\theta = \pi/2$ and find the equation for r as a function of ϕ. Show that the straight lines as a function of r and ϕ are exactly the same as those in flat spacetime).

It was this in part which caused Einstein to reject the Nordstrom theory, which predicted a metric such as the above.

2) Consider the Schwartzschild metric. Do a coordinate transformation such that $t = \tau + f(r)$. Find the functions $f(r)$ such that the spatial part of the metric is flat space. (Ie, the part of the metric which does not depend on dt is just

$$dr^2 + r^2 d\theta^2 + r^2 sin^2(\theta) d\phi^2$$

)

These metrics are called the Panlevi-Gulstrand metrics. Show that at $r = 2M$ this metric is not singular (ie does not blow up and has a well defined non-singular inverse).

Show that for the two possible signs of $f(r)$, the surface $r = 2M$ in the one metric is not the same as the surface $r = 2M$ in the other metric.

3. At what radius does the unstable circular orbit occur as a function of l and E? What is the innermost unstable circular orbit for arbitrary l and E? (This is in the Schwartzschild metric, and it is for massive particles.)

4. Consider a galaxy of mass 10^{10} times the mass of the sun. (called the imaging galaxy) on opposite sides of which we see two images of a much more distant galaxy (the imaged galaxy). Assume that one of those images of the imaged galaxy is seen to be twice as far away from the center of the imaging galaxy as is the other (all angles are a few seconds of arc). The distant imaged galaxy has a supernova go off in it. What would be the difference in times at which we would see that supernova in the two images here on earth as a function of the distance of that imaging galaxy from the earth. (assume that the distance of the imaged galaxy is many times as far away from the earth than is the imaging galaxy You can assume that the imaging galaxy is also very far from the earth.)

Note that you can assume that the images are further away from the center of the imaging galaxy than is the edge of the matter distribution in that imaging galaxy, and that the imaging galaxy can be represented as a spherical source of gravity.

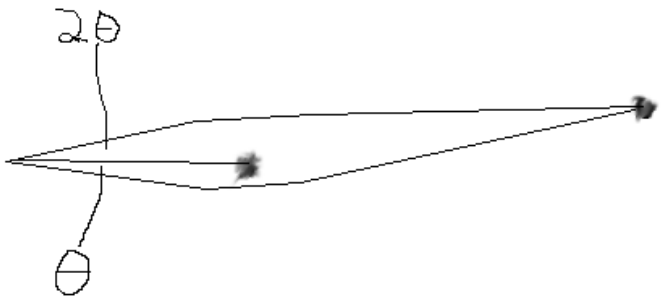

Figure 1: *Geometry of the deflection of distant galaxy light by nearby galaxy*