Physics 407-07
Assignment 5

1.) Consider the metric

$$ds^2 = -\rho^2 d\tau^2 + d\rho^2 \tag{1}$$

i)What are the Christofel symbols for this metric?

ii) What is the acceleration vector and its magnitude for the curve $\rho = \rho_0$ at various times τ?

iii)If you have a field ϕ defined at all values of $\tau,\ \rho$, what is the wave equation in terms of $\tau,\ \rho$

$$g^{AB} D_A(D_B\phi) = 0 \tag{2}$$

2.)In ordinary x,y,z,t coordinates (with metric $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$), define the antisymmetric tensor $F^{AB} = -F^{BA}$ with components (and others related to these by the antisymmetry)

$$F^{tx} = E_x \tag{3}$$
$$F^{ty} = E_y \tag{4}$$
$$F^{tz} = E_z \tag{5}$$
$$F^{xy} = B_z \tag{6}$$
$$F^{yz} = B_x \tag{7}$$
$$F^{zx} = B_y \tag{8}$$

where E_i are the usual components of the electromagnetic field, and B_i are those for the magnetic field.

i) If one has a source free Electromagetic field, show that the equations

$$D_A F^{AB} = 0 \tag{9}$$
$$D_A F_{BC} + D_B F_{CA} + D_C F_{AB} = 0 \tag{10}$$

expressed in terms of the coordinates $t, x, y.z$ are the electromagnetic field equations.

(Note– first show that for the components, the three component indicees in the second equation must all be different for the left hand side to be non-zero. Thus there are really only 4 non-trivial equations.

ii)Show that in general for an arbitrary antisymmetric $F^{AB} = -F^{BA}$, that $D_A D_B F^{AB} = 0$ Note that while the antisymmetric derivative of a scalar is assumed to be zero, you cannot make this assumption for a tensor. Instead look at the components of this tensor expression and use the properties of the Christofel symbols.

iii)Find $F^A{}_A$ and $F^{AB}F_{AB}$ in terms of E and B.

3. Show that the stress-energy tensor for the source free electromagnetic field

$$T^{AB} = F^{CA} F_C{}^B - \frac{1}{4} F^{CD} F_{CD} g^{AB} \tag{11}$$

is conserved by the equations of motion of the electromagnetic field. Ie, $\nabla_A T^{AB} = 0$.

4. The relativistic Lorentz force law for a particle of mass m and charge e can be written as

$$m \frac{Du^A}{D\tau} = e u_B F^{AB} \tag{12}$$

Show that this preserves the length of the vector u^A as it should. Show that this gives the usual force law of and electric and magnetic field on a charged particle in the non-relativistic limit in the usual flat spacetime metric

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{13}$$