Physics 407-07
Assignment 2

1.) Consider the two dimensional flat metric

$$ds^2 = dx^2 - dt^2 \tag{1}$$

and the coordiante transformation

$$x = \rho\cosh(\tau) \tag{2}$$
$$t = \rho\sinh(\tau) \tag{3}$$

Show that the curve $\rho = const$ is a timelike curve and $\tau = const$ a spacelike one.

Now take the equations

$$x = T\sinh(Y) \tag{4}$$
$$t = T\cosh(Y) \tag{5}$$

Which region of flat space-time in $x,\ t$ do these coordinates cover? What are the equations of straight lines for these equations?

2. a)Consider the set of tangent vectors t^Apointing away from the center of the earth at the surface of the earth. What are the components of this vector in r, θ, ϕ coordinates. Assume that the r component is a constant. Find the components in x, y, z coordinates.

b) consider the function $f(p) = \theta(p)$ where θ is the usual angle of that name in polar coordinates. What are the components of the cotangent vector df^B in polar and rectangular coordinates? What is the inner product $t^A df_A$ in polar and in rectangular coordinates?

3. Calculate the Newtonian potential gravitational potential for an infinite plane of matter. (assume that the solution of Newton's equations is independent of y and z) What would Einstein's guess at a metric for this distribution of matter be. Compare this to the coordinates for Rindler space given above. For the Newtonian case, does the surface $\rho = 0$ from problem 1 make any sense?

3. Consider 3 dimensional flat space in rotating coordinates

$$ds^2 = dx^2 + dy^2 - dt^2 \tag{6}$$

First define polar spatial coordinates

$$x = r\cos(\theta) \tag{7}$$
$$y = r\sin(\theta) \tag{8}$$

What is the metric in this coordinate system?

Now define a new ϕ coordinate by

$$\theta = \phi - \omega t \tag{9}$$

What is the metric in this coordinate system?

A particle at constant ϕ, r goes around in circles in the x, y coordinates. What is the gravitational potential for this particle? What are the geodesic equations for a particle in this system of coordinates? Consider the particle travelling along a geodesic. For small velocities of the particle, there is an apparent force on the particle, which depends both on position and velocity. What is that force?

(By "force" I mean the right hand side of the equations–the rate of change of coordinate velocity as a function of time.)

$$\frac{d^2 r}{dt^2} = ... \tag{10}$$

$$\frac{d}{dt}\left(r\frac{d\theta}{dt}\right) = ... \tag{11}$$

4.) Equivalence Principle:

Show that the Eotvos experiment still works even if the two masses are not the same. Ie, assume you have two objects with gravitational mass m_1 and m_2 and inertial masses of $(1+\epsilon_1)m_1$ and $(1+\epsilon_2)m_2$ hung as a torsion balance of length L from a fibre so that if they are oriented north-south, they hang horizontally and are oriented exactly North-south. If the support is rotated exactly 90 degrees, find the angle with the east-west direction that the arm of the balance hangs at, and show that this angle is proportional to $\epsilon_1 - \epsilon_2$. What is the deflection angle? (Assume that the period of torsional oscillation of the system is T seconds and that the laboratory is located at latitude θ.) You can assume that ϵ is very small and keep only first order effects in ϵ.

If the length of the torsion bar is 1 meter, m_1 is approximately m_2, and approximately 1Kg, and the torsional period is 40 min, what is the value of the deflection angle as a function of $\epsilon_1 - \epsilon_2$. Assuming that the graduate student sent into the room to measure the angles has a mass of 100Kg and is 10m from m_1 perpendicular to the torsion arm, what would be the deflection caused by the student in comparison with the deflection caused the difference in inertial/gravitational mass, if $\epsilon_1 - \epsilon_2$ is 10^{-8}. .