Physics 407-07
Assignment 1

1.) Given the metric

$$ds^2 = (x^2 dx^2 + x^4 dy^2) \tag{1}$$

find the equations for straight lines in this space.

Show that the lines $y = constant$ are solutions and find what x is as a function of s for these lines.

Change the x cordinate, so that for these solutions, the new coordinate would be a linear function of s. What would the metric be for this choice of x coordinate?

2.) For flat spacetime, where in $x,\ y$ coordinates, the metric is

$$ds^2 = dx^2 + dy^2 \tag{2}$$

define new coordinates $X,\ Y$ such that

$$x = \cosh(X)\cos(Y) \tag{3}$$
$$y = \sinh(X) sin(Y) \tag{4}$$

Find the metric in terms of X and Y (I.e., express the distance ds^2 in terms of $dX,\ dY$ and functions of X and Y). (These coordinates are called elliptical coordinates because surfaces of constant X are ellipses, and of constant Y are hyperbolas)

Are any of the lines $X = const$ or $Y = const$ straight lines? (You do not have to solve the equations but just show that they are straight lines)

3. Consider the metric

$$ds^2 = x^2 dx^2 + x^2 dy^2 \tag{5}$$

Find the geodesic equations for this metric.

Show that

$$x = iy + C \tag{6}$$

formally (for x and y complex) are solutions to these equations.

4.a) A particle of mass M at rest emits a gamma ray of energy E, leaving a particle of mass M'. What is M' as a function of E and M?

b)Now the resultant particle of mass M' in part a) at rest. What energy ϵ would a gamma ray particle need in order to be absorbed and create the original particle of mass of M again. Why is ϵ not the same as E?

5.) a) Pole and Barn revisited. A runner carrying a horizontal pole is running at a barn at 4/5 the velocity of light. When both are at rest, the pole

has exactly the same length as the barn. According to the runner, the barn will be contracted. However, when the runner, located at the trailing edge of the pole, **sees** the front of his pole hit the far inside end of the barn, how far from the end of the barn is he located?

b) Bob leaves Alice behind on earth and travels off to the nearest star 4 light years away and returns. (A light year is the distance light travels in one year) On his return, Bob finds that the total trip according to his clocks is 10 hours less than Alice claims it was. How fast was he travelling? (Assume constant velocity for the trip there and back). (Note– Use 1 year=10^4 hours.)

6.) Light traveling through water at rest travels at a speed of $\frac{1}{n}$ the velocity of light where n is the index of refraction. Assume that the water is traveling at velocity v in the lab in the same direction as the light is traveling. What, to lowest order in v is the velocity of that light according to the lab frame.