odes

I. HARMONIC OSCILLATOR

This is the name that physicists give to almost any thing which vibrates. The vibration is always a competition between two aspects– there is some mass which moves and some source of force which tries to always return the body to some equilibrium position where it can sit at rest without moving. If you displace the body from that equilibrium, that force always tries to push it back, and the greater the displacement, the greater the force pushing it back.

The oscillatory motion then comes about from a competition. If we displace the body, the force tries to push it back. This force acts on the mass, and starts it moving, the longer the force acts and the larger the force the greater the speed of motion. Once the body returns to its equilibrium position, it is moving. and a body in motion remains in motion. Thus it moves through the equilibrium point, and its displacement now in the opposite direction causes a force again pushing it back to the equilibrium. This force first slows down, and stops the body and then pushes it back toward the equilibrium again. It picks up speed, moves through the equilibrium, etc. This tradeoff of displacemtne tna motion continues– forever if there were not some other outside forces to eventually stop the motions.

It is clear that there are two aspects which determine the time it takes for the body to complete one cycle of motion. If the body is lighter,then the same force will cause it move faster and cause it to complete the cycle more quickly. If the amount of force for a given displacement is greater, then the motion and the time of the cycle will be faster. Similarly if the mass is smaller, then the given force will again move it faster. The amount of force for a given displacement I will call the stiffness. Thus the greater the stiffness, the shorter the period (the time to complete a cycle). Also the smaller the mass, the shorter the period.

As discovered by Galileo, one of the most amazing features of such motion is that it does not depend on the amplitude of the motion. The period depends only on the mass and the stiffness, nothing else. This is because the greater the amplitude the greater the force, which produces faster velocities, which makes up for the larger amplitude.

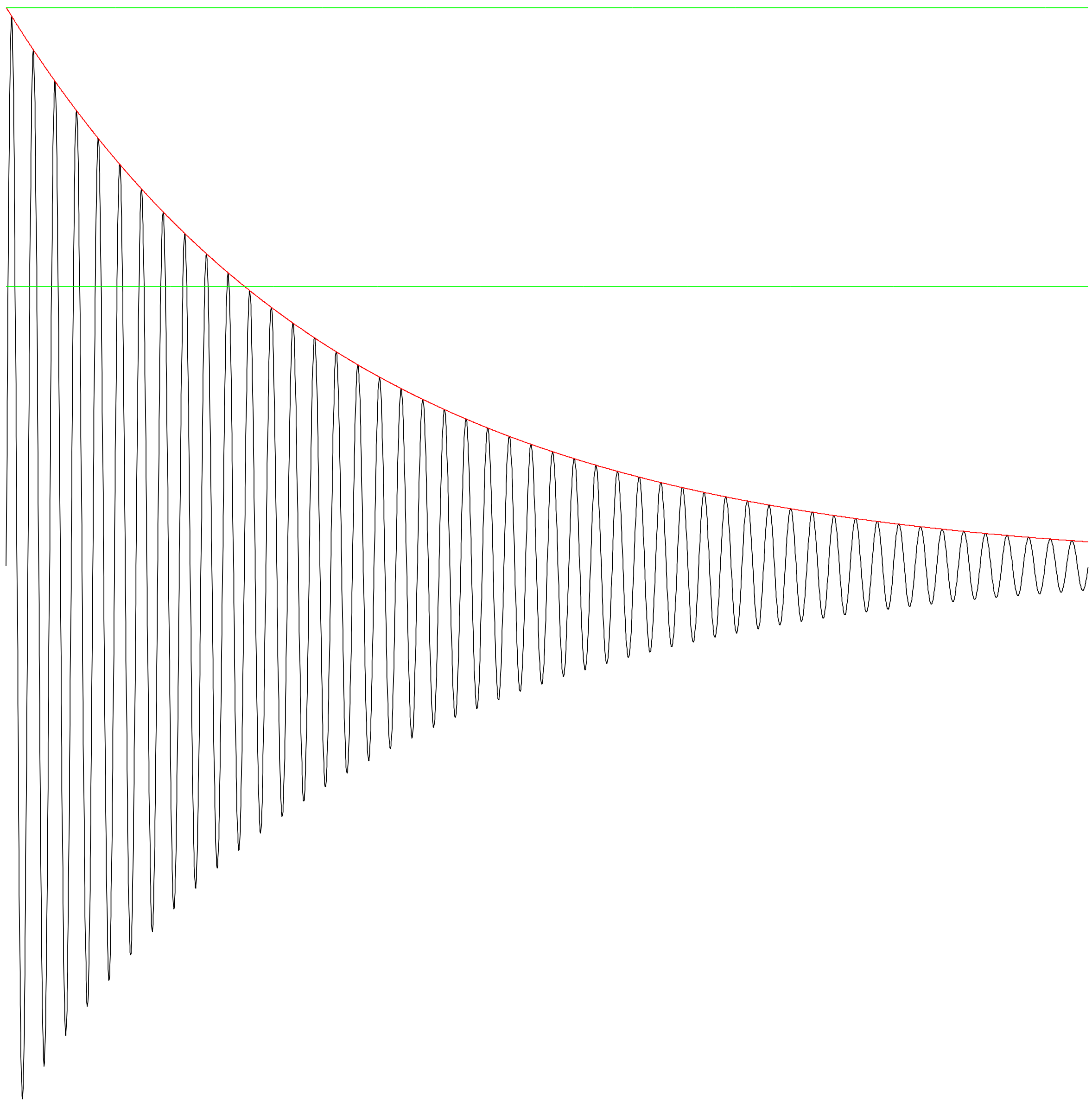
For those of you who are interested in the mathematical expression, the stiffness is usually designated by k and measured in terms of force per meter (Newtons per meter). The frequency is given by $2\pi\sqrt{\frac{k}{m}}$ where m is the mass. However, noone needs to know this expression.

II. DAMPING

For any oscillator, it will not keep oscillating forever. It gradually loses energy to the outside world, and the amplitude of the oscillation decreases. The damping depends on lots of things, but primarily with how strongly the outside world intereacts with the system. Thus, I were to immerse my apple on a hacksaw blade into water, it would intereact more strongly with the water than it does with the air, and the oscillation amplitude would decrease faster.

The measurement of the damping is designated by the letter Q (which stands for quality factor). The higher Q to smaller the damping. It is defined as 4.5 times the number of oscillations over which the amplitude decreases to 1/2 of its initial size. The 4.5 is actually $\frac{\pi}{ln(2)}$ and arises from the mathematical definition interms of radians and e, the base of the natural logarithms which physicists and mathematicians like to use, but it completely irrelevant for us. Eg, if a vibration of 1000Hz decays to half its amplitude in 1/10 of a second, then there are 100 oscillations in that time, and the Q would be $4.5 \times 100 = 450$. An oscillator with a high value of Q takes a long time to decay, while one with a small value decays very quickly.

In the figure we have an example of the graph of the oscillation of some oscillator. The black curve is the plot of the instantaneous position of the oscillator at each time, just as with the graph we got for the “human oscilloscope”. The red curve gives a smooth curve designating the maximum amplitude over time. The two green curves give the maximum and half maximum amplitudes. The intersection of the lower green curve with the red curve is that point at which the amplitude has fallen to half its initial value. This is about 11 cycles of the oscillator. Thus the Q value for this oscillator is approximately 4.5 times 11 or about 50. (Note that one almost never needs to quote the Q value to better than about 1 significant figure– ie only the first figure of the number is important)



(Values of Q less than 1 can even be defined. Measuring them has to be done by a procedure different from counting cycles)

III. RESONANCE

If an oscillator is shaken by some outside force, it will respond by moving, but how much it moves depends on the frequency of the outside force. As you can imagine, if you have a child on a swing, if you push very very slowly– ging one tiny push over a time much longer than the swinging freqency– the response will be very small. Similarly if you push time and again very rapidly, there will be little response. Howerver if your pushes occur on the same time period as the swinging, the response can get very large, even if the pushes are very small. Ie, the amplitude of the response depends crucially on the frequency of the pushes. The maximum response occurs when the frequency of the pushes is exactly the same as the natural frequency of oscillation of the system. This is called resonance.

The amplitude of the response at resonance depends on the Q of the oscillator. The higher the Q, the larger the response (If the child on the swing drags her feet on the ground on every swing, making the Q very small, it is very hard to get her swinging with a very high amplitude. On the other hand if the child doesnot, the swinging has a high Q, and the amplitude to which you can get her swinging even with small pushes can be very high.

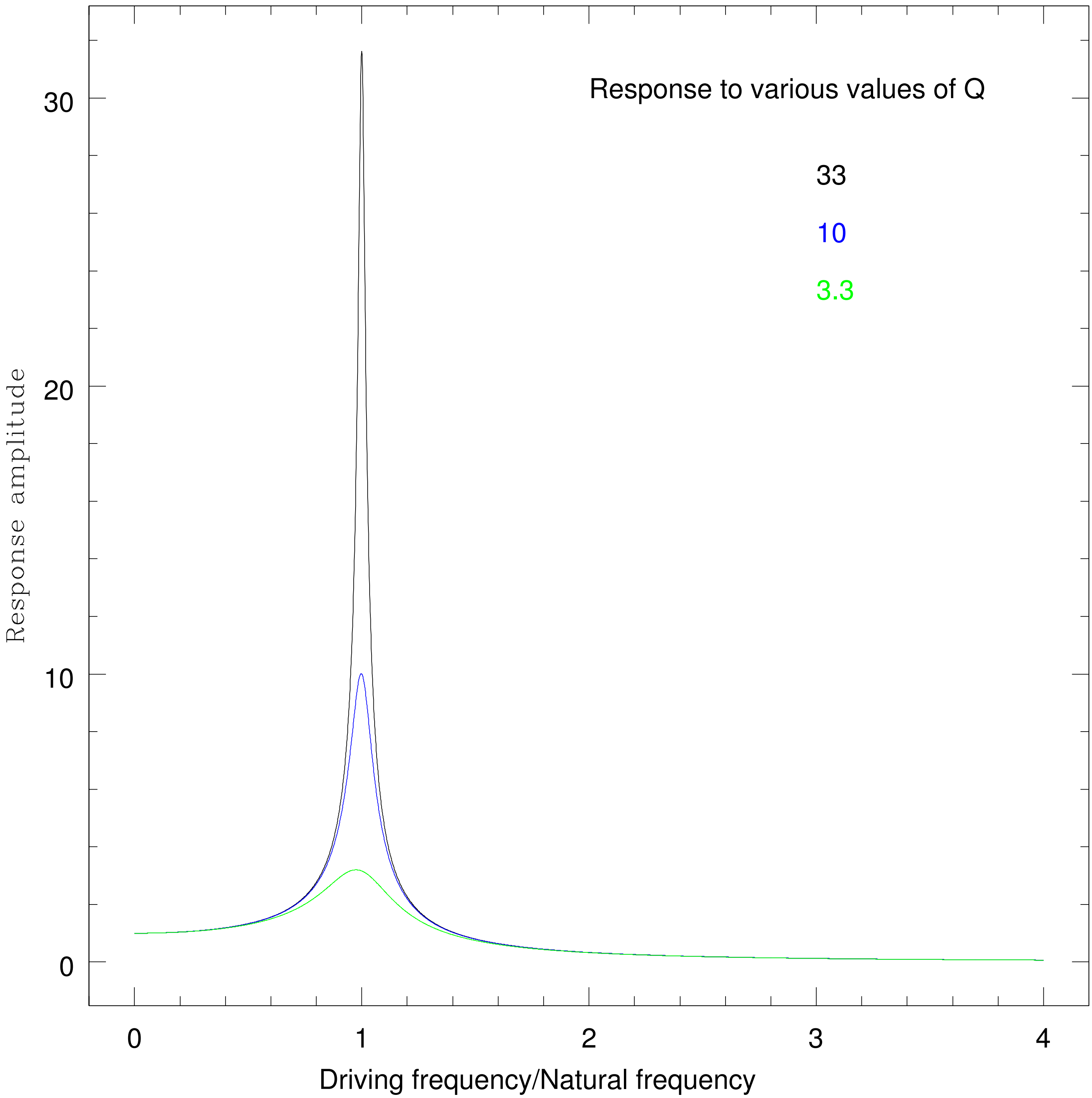


Figure 1 is a graph of the amplitude of the response vs the frequency at which the pushing is done. The high peak occurs at the naturual frequency of the oscillator. The height of the peak is proportional to Q. Further more the width of the peak (usually determined by the fractional width of the peak at an amplitude which is half the maximum– full widthe half maximum) is proportional to 1/Q. Ie, the greater the response of the oscillor, the more accurately you have to tune the frequency of the outside source to the natural frequency of the oscillator in order to ge a large response. Furthermore, the longer it takes to come up to that maximum oscillation amplitude. Ie, it takes a time of order Q for the oscillator to come up to the maximum amplitude.

For driving frequencies far from the resonant frequecy (natural frequency) the response is roughly independent of Q but for frequencies near the resonant frequency the response can depend critically on Q. If Q is small (lots of damping) the response tends to be small. If Q is large (little damping), the response depends crucially on Q