Behaviour of multiple and stiff strings for each note on piano

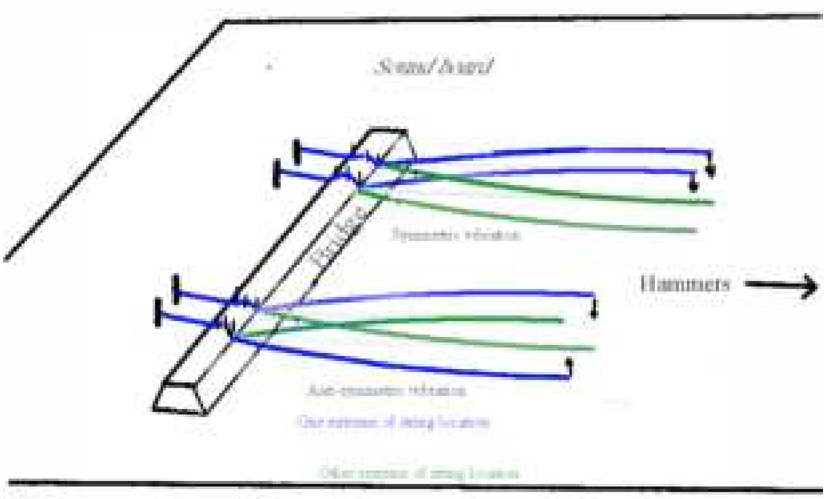
I. MULTIPLE STRINGS PER NOTE

In order to make the piano loud, each pitch is played not by one string but by 2 or three. In a grand piano, the lowest1/4 are single string, with stainless steel piano wire clad with a spiral of metal, like brass to make the string heavier and giving a lower pitch for the same tension. The two strings are tuned to the same frequency, but they are tuned with the other of the two strings held stationaryand damped. Now one might expect that this would ensure that both strings were tuned to the same pitch when the hammer strikes them both. however when both strings are vibrating in the same direction, the pitch is slightly lower than each string on their own since they exert a larger force on the bridge, causing the bridge to vibrate in the same direction as the strings are pulling.This increases the moving mass and thus lowers the pitch (slightly) of the two strings vibrating together each in the same direction. Since now both strings are pulling on the bridge it cases a greater motion of the bridge and thus of the soundboard of the piano, making the energy radiated 4 times greater than with one string alone.

There is however another way the strings could vibrate, and that is opposition to each other. Thus the two strings do move the bridge in the same way as if they were oscillating symmetrically. But because the two stings are pulling up-down on the bridge at slightly different places, they cause the bridge to twist. The bridge is a long piece of wod, and so it does not twist much, and thus the soundboard does not vibrate much either. This means that this antisymmetric motion will be quieter and will also transfer less of the vibration of the string to the soundboard.

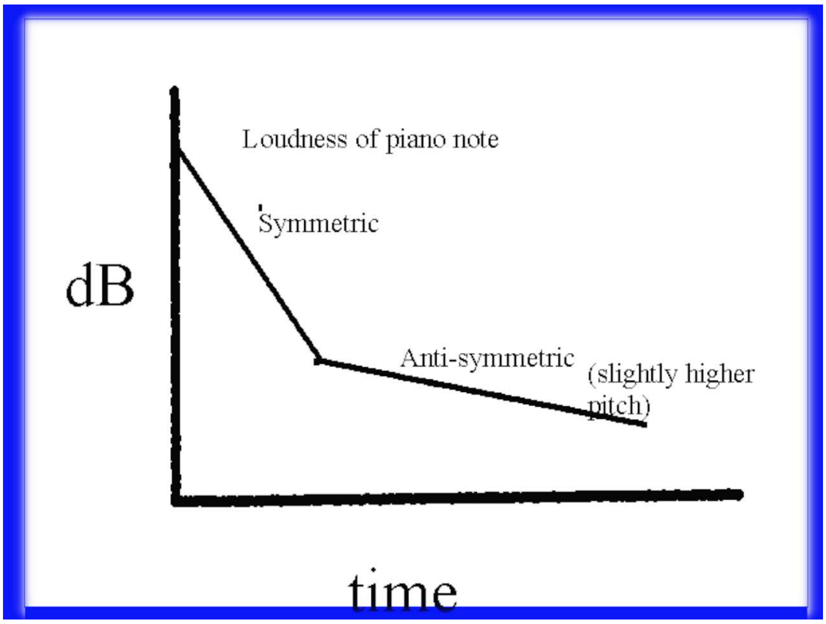
The primary way that the strings are damped (loose energy) is because of the conversion of the oscillation of the strings to motionof the soundboard, and the production of sound.

Thus if you hold down the key of the piano so the string is not damped by the dampers, you will hear the sound intensity decay. This is primarily the symmetric mode. Eventually it will have the intensity of the anti-symmmetric mode, which damps out more slowly due to sound emission by the antisymmetric mode being less. Thereafter one will hear the antisymmetric mode, whose frequency is slightly higher than that of the symmetric moce, and whose decay is slower.



The two modes of motion of a piano note with two strings. This shows both the symmetric mode (which pulls and pushes on the bridge with much larger amplitude than the anti-symmetric mode., and thus is much louder and slightly lower pitch than the anti-symmetric mode.

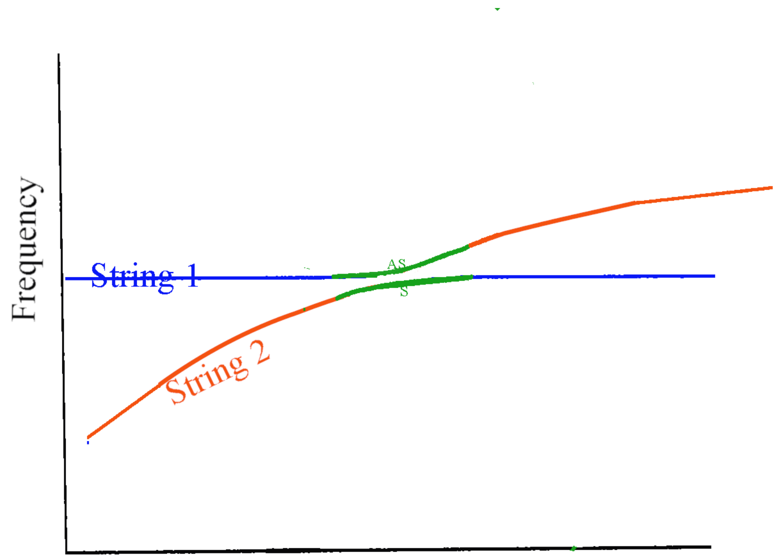
The change in decay of the note, with the intial rapid decay being due to the symmetric mode, until the symmetric mode has the same amplitude as the anti-symmetric mode, when it takes over the sound and decays more slowly



The fact that the even though the two strings are each tuned to exactly the same frequency, thei symmetric and anti-symmetric modes have different frequencies is known in physics as "avoided level crossing". This is phenomenon which is well known in quantum mechanics, especially in atomic physics. Atoms have discrete frequencies for the transitions from one state to the other, just as strings have discrete frequencies for the modes of a string. In the case we are looking at, we can draw the frequencies for the symetric and antisymmetric behaviour of the two strings vibration frequencies.

One can plot the frequencies of the modes of the two strings, as for exmple, one changes the tension in one of the strings (lets call it the second one), leaving the tension for the other string alone. As one changes the tension in the string2, one expects the frequency of that string to cross the frequency of the other. (the two frequencies be equal)

Instead what happens in as in the next figure.



Ie, the frequencies of the two strings never cross. Instead the modes go from the two separe string each vibrating, while the other stays almost still, to the two modes being the symmetric and anti-symmetric modes, to the two strings being separate again but never crossing (the frequencies of the two modes of vibration are never the same.) Thus in this feature a piano is like say, a Hydrogen atom, where one changes the frequencies, not by altering some tension, but rather alters for example the magnetic field that the atoms are placed in. But the discrete frequencies behave just like the discrete frequencies of the piano strings as one changes the tension of the strings.

So, one has a pub question: How is a piano like a Hydrogen atom?

Both have an avoided level crossing as one changes a parameter (frequency for the piano, and magnetic field for the H atom)

Mind you you might just get a blank stare and a "HUH?".

In the highest range of the piano, the strings are often trippled, i.e., three strings per note. Now there are three modes of the strings. The Symmetric one is the same as for two– namely all strings oscillating in the same way together. This makes them louder by about 10dB then if there was just one string. but also flattens the note. The second mode has the central string not vibrating, but the two outside strings oscillating out of phase with each other, just like the anti-summetric mode of the two string situation. Finally one has a mode where the two outside strings are oscillating in phase with each other, but the inside string oscillating with twice the amplitude and 180 degrees out of phase with the two outside strings. The highest strings damp out very rapidly because the strings are so short, their thickness and their mass so low, so the amount of energy in the vibration of the string is small. The frequencies of the modes will have the symmetric vibration the lowest, the outside antisymmetric mode higher and the third mode the highest. Again these are relatively small effects.

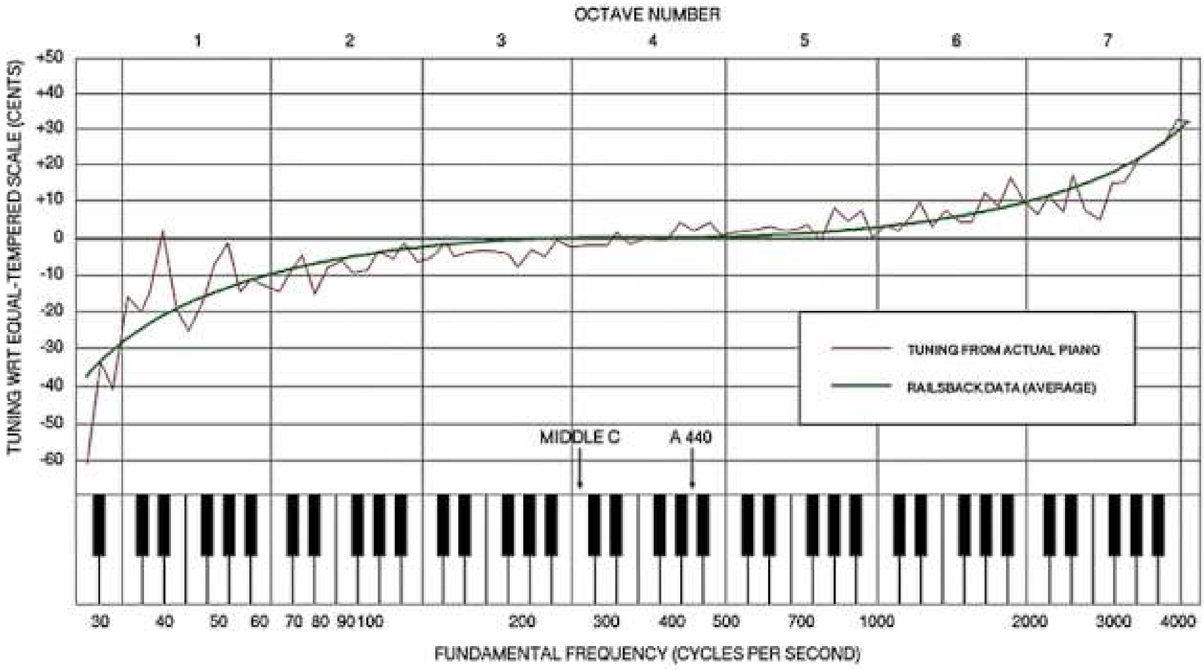
II. STRING STIFFNESS

If the piano string is made of a high stiffness material (as piano wire, which is used on the piano, is) then the restoring force (the force pushing the string back toward its rest position) is not just due to the tension in the string, but also due to the stiffness of the string. This stiffness tries to straighten out the string much more so than does the tension, and this means that the frequencies of the modes rise faster than just the mode number. For a floppy string, the frequency of the second mode is twise tha tof the first mode, the frequency of the third is three times, the fourth is four time, etc. The stiffness of the string has an effect which goes as the square of the mode number For example, if the stiffness raised the frequency by .1%, the second mode would have its frequency raised by .4%, the third by .9%, the fourth by 1.6%, etc. Ie, the frequencies are not integer multiples of the fundamental, but get progressively sharper, the higher the mode number. That means that if one plays a key of some pitch, and another an octave higher, the second mode will by sharper than that octabe, the fourth mode would be even sharper than two octaves, etc. Thus the if one played two notes an octave apart, one would get beats between the varous modes of the notes. This would result in two notes an octave apart sounding harsh and out of tune with each other. But the octave (and two octaves as well) are supposed to be the most harmonious pair of notes. The piano is out of tune with itself, and there is nothing one can do to make them harmonious. This gives the piano an edginess that no other instrument has.

This effect is smaller if the string is longer– thus the large size of a concert grand piano. Piano tuners tuning home pianos, which are usually upright, whose strings are relatively short, and this inharmonicity is large, so that octaves sound more in tune with each other than you would get if you make the octaves twice the frequency of the lower note. The hiher octaves get more and more sharp of the proper twice frequency.

This also means that if the piano plays with other instruments (eg, in a concerto where the piano plays with the other instruments in the orchestra), if it is tuned to be most in harmony with itself, it is out of harmony with the rest of the instruments.

Piano tuners try to compensate for this feature of the strings. In particular in the bass strings they tend to tune them flatter, but up to a half a semitone, in order that when one plays a bass string in chords with pitches near the middle of the keyboard, the bass string does not sound too out of pitch with the central pitches. Likewise the tune the top strings sharp, again by up to half a semitone, in order that they not be too dissonant in chords with the central notes. Near the center they tune the octaves as pure so that more typical chords in which many times one has octave of fifth mirroring, one does not get to discordant a sound. Here is a graph (from at least 50 years ago) which plots the tunig curves of a single piano, and the average tuning of a large number of pianos.



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Is this a good thing or a bad? We have gotten used to it, and the piano thus produces edgier music than most other instruments, and may well be part of the driving force behind 20th century music being more dissonant than earlier music.

Also the fact that it is out of tune withitself tends to get hidden in the equal temperament tuning of the piano (in fact equal temperament came in with the piano in the mid 19th century.) However it adds to the problems of equal temperament in that the not only are internals like the major and minor thirds, and fourths and fifths, somewhat out of tune with what the brain would like to hear, but even the octaves are out of tune because of the stiffness of the strings

Remember that this all arises because you want the piano to be loud, and thus has to use multiple strings and strings which can withstand a lot of tension and are thus also very stiff.

Note that this is also a problem on an electric guitar, where the strings are made of steel with significant stiffness, meaning that the guitar is always out of tune with itself, even on the octaves, and it is worse than on the piano because the strings are much shorter. (A short string is worse because the curvature of the string is much larger for a given note, bringing in the an-harmonicity of the stiff string to a greater extent. But then again, does one want an electric guitar to sound sweet and soothing, or does one want it sounds like wolves howling?

. Acoustic guitars, or lutes, if tuned with floppy strings (eg nylon or dacron or gut), do not suffer from this. But they are also not very loud.