The effect of the reed resonance on woodwind tone production

Stephen C. Thompson[a)]

Case Western Reserve University, Cleveland, Ohio 44106
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In normal woodwind tone production the nonlinear flow control properties of the reed transfer energy among the harmonics of the spectrum, and the favored playing frequency is one for which the air column input impedance is high at several harmonics. Above the middle of the second register, woodwinds have only one participating impedance peak; yet these notes can be played even without the use of a register hole, despite competing possibilities of low register intermode cooperation. Such notes are possible because enhancement of the reed's transconductance A near its own resonance frequency can offset the small input impedance Z of the air column so that $(ZA-1)>0$, providing an additional means for energy production above cutoff. Spectral levels as a function of blowing pressure, air column impedance, and reed characteristics are derived. Experiments on the clarinet show that the player can adjust the reed resonance frequency from about 2 to 3 kHz. When the reed frequency is adjusted to match a harmonic component of the tone, the amplitude of that component is increased, and the oscillation is heard as being stabilized in loudness, pitch, and tone color.

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INTRODUCTION

This paper presents a further development and extension of the theory of nonlinear self-sustained musical oscillators which was initiated by Benade and Gans[1] and formalized by Worman.[2] In its present form the theory can now describe the steady-state oscillation mechanism and general behavior of all notes of the clarinet. Most of the conclusions also apply to other reed instruments and, with certain modifications, to the brass instruments as well.

Some of the earliest work which has direct application to reed woodwinds is that done by Weber[3] in about 1830. While his work was mainly concerned with metal reeds on organ pipes, Weber calculated the natural frequency of an air column terminated by a reed. He did not deal with the regeneration mechanism required to maintain the oscillations, although he observed experimentally that certain combinations of reed and air column natural frequencies would not sustain an oscillation. About 30 years later, Helmholtz[4] showed that the regeneration mechanism places restrictions on the relative phase of the oscillations of the reed and air column which can only be satisfied if the playing frequency is either very near the natural frequency of the reed or slightly below the natural frequency of an air column mode. In 1963 John Backus presented[5] a linear theory of clarinet oscillations which was valid for very small amplitudes near the threshold of oscillation. He was able to calculate the threshold blowing pressure and the oscillation frequency at threshold. Of even more importance to the present work, Backus also made measurements of the flow into the air column as a function of reed tip opening and pressure difference across the reed. As expected, he found that the flow is a nonlinear function of both variables, and he determined the form of this function. Nederveen extended Backus' theory to the double reeds and presented measurements which show that the flow through double reeds is a different nonlinear function of the pressure difference and reed opening,[6] but he also did not include the effects of the nonlinearity in the theory. Fairly recently, still another small-amplitude linear theory has been presented[7] by Wilson and Beavers. None of these theories are capable of explaining the amplitude and spectrum stability which are present in all musical oscillations. This stability can only be explained by including the nonlinearities.

In 1929 Henri Bouasse published[8] the results of his work on wind instruments. Bouasse understood very well the role of the reed in maintaining oscillations. While he did not develop any new mathematical theories, he presented many observations which anticipate the nonlinear theory which has recently been developed. For example, he stated[8] without explanation the fact that

"the maintenance of the standing wave is faciliated by the coincidence of the qth harmonic of the pressure spectrum due to the (nonsinusoidal air flow through the reed) with the tube mode whose frequency is $N=qn$ (where n is the playing frequency). In this case the (flow) becomes very strong and very stable: One recognizes that the reed motion will be stabilized for the frequency n."

The first attempts to include nonlinear effects in the explanation of musical instrument tone production were made in 1958 by Benade[9] in a series of reports for C. G. Conn LTD. This work was continued by Benade and Gans[10,11] and its continued development has led to the present work. In a paper[12] delivered to the Acoustical Society, Pyle presented a different nonlinear theory of brass instrument oscillations whose initial results looked promising. Fletcher[13] has developed a similar theory for organ flue pipes which is able to predict both the transient and steady-state pressure spectra in good agreement with measured spectra. His theory depends

[a)]Current address: Acoustics Department, Gould Inc., Ocean Systems Division, 18901 Euclid Avenue, Cleveland, OH 44117.

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upon the fact that the flow into the flue pipe is controlled by the velocity of the standing wave at the mouth of the pipe. It cannot be applied to the reed instruments whose flow is controlled by a pressure operated reed. Quite recently Schumacher[14-16] has developed an integral equation theory to predict the steady-state behavior of the bowed-string instruments and has extended it to include both organ flue pipes and reed instruments. His initial results agree with those of other workers, and the method shows promise of developing new insights into the problem.

Worman formalized the Benade–Gans theory for those reed instruments for which the Bernoulii force on the reed tip can be neglected. He was able to solve the coupled nonlinear algebraic equations of the theory for a single simple case to show the validity of the theory. Benade[17,18] has applied the same general method to the oscillations of the brasses and the bowed strings as well as extending its application in woodwinds. The theory has been successful in explaining many aspects of musical instrument behavior which had not been explained by earlier linear theories. This, and many other results of Worman's theory, are presented clearly and with very little mathematics in Benade's *Fundamentals of Musical Acoustics*.[19]

Throughout the development outlined so far, all investigators have assumed that the natural frequency of the reed is sufficiently high above the playing frequency that the reed resonance does not play an active role in the regeneration process. The possibility of oscillation just below the reed frequency has long been recognized and Bouasse discussed musical oscillations of this type. However, for normal musical oscillations based on an air column mode, it has usually been assumed that the reed frequency is far above the playing frequency. The present work shows that in fact, the reed resonance can play a dynamically significant role in maintaining the oscillations in the upper registers of reed instruments when the reed frequency is adjusted to match the frequency of a low-order harmonic multiple of the playing frequency.

Along somewhat different lines, Bariaux[20] is developing a method of solution for reed instruments which holds only when the reed beats and is rigidly closed for a part of the cycle. This is very important in understanding the many instruments whose reed beats at low playing levels; among them are the bassoon, the oboe, and the clarinet with a French style mouthpiece and reed. In addition, the reeds of all instruments beat at high playing levels. The theory developed in the present work does not hold when the reed beats. The initial results of Bariaux for the beating reed show many similarities with the results presented here for the nonbeating reed.

I. THEORY

The theoretical treatment presented here is similar to that developed[2] by Worman; however, a major emphasis is placed on the reed characteristics, which played only a peripheral role in the earlier work. In all cases the notation is chosen to match that used by Worman. This paper deals specifically with an idealized clarinet-like system, although many of the results apply to all reed instruments for which the Bernoulli force on the reed tip (due to the air flow through the reed) can be neglected. It is reasonable to postpone consideration of the Bernoulli force at this time because acceptable musical instruments can be made in which the force is negligible, although small changes in this force, produced by altering the dimensions of the mouthpiece profile, are readily perceived by the player and can have considerable musical significance. The present formulation does not accurately describe the double reed instruments because their reeds are strongly influenced by the Bernoulli force. However it is known that many generalizations from the present work do apply to the double reeds. The model clarinet system used in the theory is composed of a reed mounted at one end of a particular musical air column. The reed and air column are both assumed to behave as damped linear oscillators which are coupled because the mouthpiece pressure provides the driving force for the reed motion, while the air flow through the reed adds energy to the oscillations in the bore. This air flow, and thus, the coupling it provides, is a highly nonlinear function of both the pressure difference across the reed and the reed tip opening. The reed thus plays a dual role in the model. It acts as a linear oscillator at the end of the air column driven by the pressure variations in the mouthpiece, and it also serves as a nonlinear flow control valve which can add energy to the oscillation of the air column.

The assumption that the reed behaves as a linear oscillator introduces the restriction that the reed motion must not be so large that the reed beats against the tip of the mouthpiece. In practice, for clarinet reeds and mouthpiece designs normally used by orchestral players in the United States, such beating takes place only at high playing levels. If we look a little more closely at the motion of a clarinet reed, we find that it may not behave as a linear oscillator even when it is not beating. Explicit introduction into the theory of the nonlinearity of the reed dynamics, on top of the nonlinear flow control characteristic, would vastly complicate the mathematical analysis without making any changes in the general behavior of the equations. Various coupling coefficients would be changed, but since the nonlinearity in the flow control characteristic of the reed is much larger than that in the reed response, these changes can be thought of as perturbations. Furthermore, observations by Backus of reed opening versus pressure difference across the reed show that, at least far below its resonance frequency, the reed acts[21] very much like a linear oscillator driven below resonance. For these reasons, the reed is treated as a linear oscillator. The oscillations of the air column are also assumed to be linear, and here the approximation is much easier to justify. The only major source of nonlinearity in the air column of musical instruments is turbulence at the sharp corners at the edges of tone holes and in the joints of the instruments. By carefully rounding these sharp corners, the turbulence effects can be minimized so that they only become important at high playing levels where the other assumptions of the theory also fail.

If the reed is assumed to be a damped linear oscillator driven by the periodic pressure difference across it, the differential equation for the displacement of the reed tip, y, is

$$\frac{d^2y}{dt^2}+g_r\frac{dy}{dt}+\omega_r^2 y=-\frac{1}{\mu_r}p\,, \tag{1}$$

where the reed is characterized by its resonance angular frequency ω_r, half-power bandwidth g_r, and effective mass per unit area μ_r. The pressure difference encountered in crossing from the outside to the inside of the reed is p, and the negative sign occurs because a positive pressure difference tends to close the reed. Solving this equation for the sinusoidal excitation $p = Ae^{j\omega t}$ leads to:

$$y=-p/[\mu_r(\omega_r^2-\omega^2+j\omega g_r)]=-D(\omega)p\,, \tag{2}$$

$$D(\omega)=de^{i\delta}\,, \tag{3a}$$

$$d=\{\mu_r[(\omega_r^2-\omega^2)^2+\omega^2 g_r^2]^{1/2}\}^{-1}\,, \tag{3b}$$

$$\tan\delta=-\omega g_r/(\omega_r^2-\omega^2)\,. \tag{3c}$$

$D(\omega)$ is the complex reed response coefficient whose magnitude and phase are d and δ respectively.

The air column chosen for the theoretical study is similar to that of a clarinet. The basic air column parameter which enters the theory directly is the bore input impedance Z_b—the ratio of acoustic pressure-to-volume flow at the tip of the mouthpiece where air enters the air column. The incoming flow divides into two parts, one entering the air column and the other filling or leaving the space occupied by the reed as it swings back and forth. The pressure which drives the flow into each of these regions is the pressure within the mouthpiece. An impedance can be defined for each of these parts of the flow, and since the pressure associated with each is the same, the total input impedance Z of the air column is the "parallel" combination of the input impedance of the air column Z_b and the impedance associated with the reed Z_r.

$$\frac{1}{Z}=\frac{1}{Z_b}+\frac{1}{Z_r}\,. \tag{4}$$

Typical bore input impedance curves for a note of the clarinet are shown in Fig. 1. For simplicity, all calculations in this paper are done on the assumption that the input impedance of the bore beyond the tone hole cutoff frequency is strictly constant and equal to the characteristic impedance of the tube. For comparison with these calculations, a clarinet-like system was built which has a very flat impedance beyond cutoff. The next section of this paper describes the musically important case in which one or another small impedance peak beyond cutoff can become a significant part of the oscillation mechanism if it falls near the reed resonance frequency where the reed transconductance is quite large.

The acoustic impedance associated with the flow into the region behind the reed is found by calculating this flow and dividing it into the mouthpiece pressure. The required volume flow is just the volume per unit time swept out by the reed as it swings back and forth. If

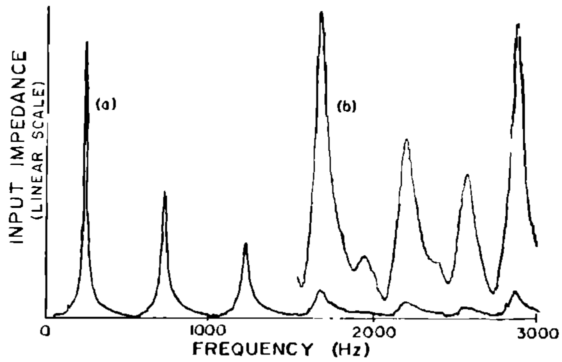

FIG. 1. (a) Measured input impedance of B♭ clarinet playing the note written C_4. (b) Same as (a) but enlarged ×10 showing details of impedance beyond cutoff.

w is the width of the reed, x is a coordinate which measures position along the reed from the reed tip, and $Y(x)$ is the reed displacement from its equilibrium position at the position x, then the acoustic volume flow associated with the reed motion is

$$u_r=w\int\frac{dY}{dt}\,dx\,, \tag{5}$$

where the integration extends over the entire moving length of the reed. It is assumed that all points on any line perpendicular to the length, are equidistant from the facing and move in phase. For sinusoidal excitation, this may be written

$$u_r=w\int\frac{dY}{dt}\,dx=S_r\frac{dy}{dt}\,, \tag{6}$$

where S_r is an effective reed area and y is the displacement of the reed tip from equilibrium. Combining Eqs. (2) and (6) to find u_r, the reed impedance is found to be

$$Z_r=-\frac{p}{u_r}=-\frac{p}{S_r dy/dt}=\frac{p\mu_r(\omega_r^2-\omega^2-i\omega g_r)}{-i\omega p S_r}$$
$$=\frac{\mu_r}{\omega S_r}[\omega g_r+i(\omega_r^2-\omega^2)]\,. \tag{7}$$

According to Eq. (4), this impedance is in parallel with the input impedance of the bore to yield the total input impedance of the air column. Figure 2 shows an idealized typical air column input impedance curve for a cylindrical instrument such as a clarinet. The dip in the total impedance at the reed natural frequency is caused by the decreased reed impedance in this frequency range. This impedance curve is used in the theoretical discussion.

We now use Backus' expression for the acoutic volume flow through the reed aperture which, rewritten in terms of the present notation, is

$$u=Bp^{2/3}(y+H)^{4/3}\,, \tag{8}$$

where p is the pressure difference across the reed, H is the equilibrium opening of the reed tip, y is the reed tip displacement from equilibrium, and B is a dimensional constant whose value is 0.08 SI units. Backus' experiments were carried out under nonoscillatory conditions and thus the effects of the inertia of the air

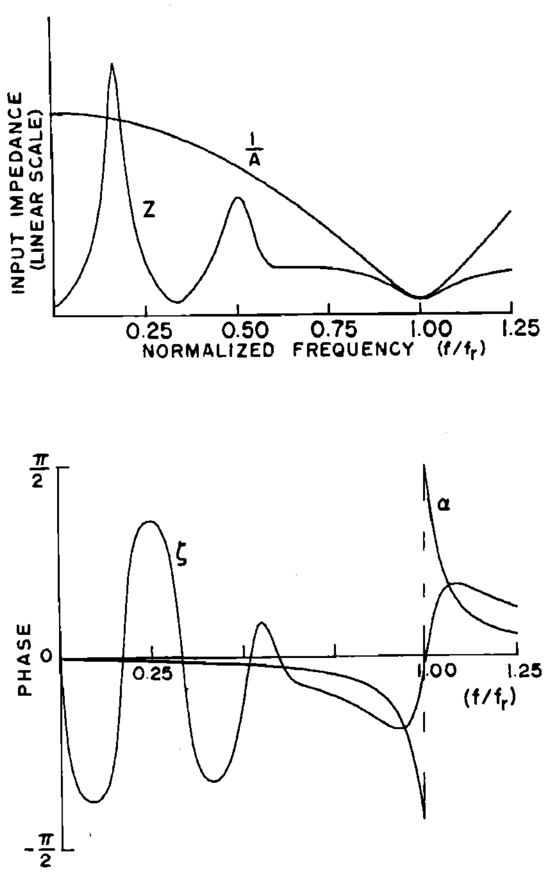

FIG. 2. Magnitude and phase of air column input impedance (Z, ζ) and minimum impedance for oscillation $(1/A, \alpha)$. Oscillation is possible when $1/A \geq Z$ and $\xi = \alpha$.

mass in the reed opening are neglected. However, because the reactance due to this inertia is much smaller than the acoustic resistance implied by Eq. (8) at all frequencies of interest, this relation is used without correction.

Equation (8) is expanded in a two-dimensional Taylor series about some appropriate values of p and y, and coefficients of like powers of p and y are collected to yield

$$u = \sum_{i,j=0}^{\infty} F'_{ij} p^i y^j . \tag{9}$$

Next the relationships between pressure and flow of Eq. (10), and pressure and reed displacement of Eq. (2), are used to eliminate u and y from Eq. (9). The mouthpiece pressure is the product of the air flow into the air column and the input impedance Z, whose magnitude and phase are z and ζ. The mouthpice pressure is the difference between the blowing pressure P and the pressure difference across the reed p. Thus

$$(P - p) = Zu = ze^{-i\zeta}u . \tag{10}$$

The reed response function $D(\omega)$, which relates the reed displacement to the pressure difference across the reed, is defined in Eqs. (2) and (3). It must be remembered that D and Z are defined only for sinusoidal excitation, and thus for the musical case where the variables contain several harmonically related frequency components, Eqs. (2) and (10) must be considered as operator equations.

For a periodic oscillation, each of the variables u, p, and y may be expanded in a Fourier series.

$$p = \sum_{n=0}^{\infty} p_n \cos(n\omega t + \phi_n) , \tag{11a}$$

$$u = \sum_{n=0}^{\infty} u_n \cos(n\omega t + \psi_n) = \frac{P}{z_0} - \sum_{n=0}^{\infty} \frac{p_n}{z_n} \cos(n\omega t + \phi_n + \zeta_n) , \tag{11b}$$

$$y = \sum_{n=0}^{\infty} y_n \cos(n\omega t + \chi_n) = \sum_{n=0}^{\infty} p_n d_n \cos(n\omega t + \phi_n + \delta_n) . \tag{11c}$$

Here $n\omega$ is the nth harmonic of the playing angular frequency ω, and the subscript n signifies that the variable is to be evaluated at the frequency $n\omega$. Equations (2) and (10) have been used to express u_n and y_n in terms of p_n. Equations (9) and (11) can now be combined to yield a single equation for the amplitudes of the Fourier components of the pressure difference across the reed.

$$\frac{p}{z_0} - \sum_{n=0}^{\infty} \frac{p_n}{z_n} \cos(n\omega t + \phi_n + \zeta_n) = \sum_{i,j=0}^{\infty} F'_{ij} \left(\sum_{n=0}^{\infty} p_n \cos(n\omega t + \phi_n) \right)^i \times \left(\sum_{n=0}^{\infty} p_n d_n \cos(n\omega t + \phi_n + \delta_n) \right)^j . \tag{12}$$

To study the general nature of the oscillations, it is necessary to retain only the first few terms, and thus in this discussion the Fourier series is terminated with $n = 3$. Detailed numerical calculations would require keeping more terms. In the present treatment, the Taylor series is terminated with $i = j = 2$. As with all series approximations, keeping only the first few terms is expected to give an acceptable approximation only at small excitation amplitudes. In the present case, however, the second order approximation has been found to give at least qualitative agreement with experiment at all amplitudes for which the reed does not beat against the tip of the mouthpiece.

At this point the products indicated in Eq. (12) are expanded. Because of the linear independence of sines and cosines of different frequencies, the resulting equation can be divided into a set of coupled nonlinear equations each of which contains the coefficients of a single-frequency sinusoid from Eq. (12). This set of equations can be written in the form which appears below.

$$P - p_0 = G_{01}p_0 + G_{03}p_0^2 + G_{04}p_1^2 + G_{05}p_2^2 + G_{06}p_3^2 + G_{07}p_0^3 + G_{08}p_0p_1^2 + \cdots , \tag{13a}$$

$$p_1 = \frac{a_0p_0p_2 + a_1p_0p_3 + a_2p_2p_3 + \cdots}{(\cos_1/z_1) - A_1(d_1, \delta_1)} = \frac{b_0p_0p_2 + b_1p_0p_2 + b_2p_2p_3 + \cdots}{(\sin\zeta_1/z_1) - A_2(d_1, \delta_1)} , \tag{13b}$$

$$p_n \cos\phi_n = p_1^n B_n(d_1, \delta_1)\{[\cos\zeta_n/z_n - A_1(d_n, \delta_n)]^2 + [\sin\zeta_n/z_n - A_2(d_n, \delta_n)]^2\}^{-1/2} , \tag{13c}$$

$$p_n \sin\phi_n = p_1^n C_n(d_1, \delta_1)\{[\cos\zeta_n/z_n - A_1(d_n, \delta_n)]^2 + [\sin\zeta_n/z_n - A_2(d_n, \delta_n)]^2\}^{-1/2}. \quad (13d)$$

The A_i, B_i, and C_i in these equations are to be considered as constants whose values depend on the reed parameters, the frequency, and the nonoscillatory pressure component p_0, but not on the amplitudes of the oscillatory components. At higher amplitudes it is necessary to consider terms beyond the second order in the Taylor series expansion. In that case the A_i, B_i, and C_i contain arbitrary powers of all of the p_i. A_1 and A_2 are actually the real and imaginary parts of the reed transductance.

One notices that one possible solution to Eqs. (13) is $p_i = 0$, $i = 1, 2, 3, \ldots$. This nonoscillatory solution is possible for any value of the blowing pressure P. As p_0, the nonoscillatory pressure difference across the reed is increased from zero; the only way for an oscillation to start is for the denominators of both of the expressions in Eq. (13b) to be simultaneously zero. It is convenient in this discussion to use the magnitude and phase of the reed transconductance rather than its real and imaginary parts. Thus we define

$$A = (A_1^2 + A_2^2)^{1/2}, \quad (14a)$$

and

$$\alpha = \tan^{-1}(A_2/A_1). \quad (14b)$$

The requirements for an oscillation to begin are then

$$\alpha = \zeta_1, \quad (15a)$$

and

$$1 - z_1 A = 0. \quad (15b)$$

These express the familiar criteria for a linear feedback oscillation.[22] Reference to Fig. 2 shows that these requirements are met in two frequency regions. Oscillation is possible at a frequency slightly less than the frequency of a peak in the input impedance, and also at a frequency slightly less than that of the reed resonance.

The reed damping used to calculate A in Fig. 2 is somewhat less than that under actual playing conditions. The damping provided by the lip is large enough that ordinarily the curves of Z and $1/A$ do not cross near ω_r. Thus the "reed regime" oscillation near the reed frequency cannot usually be obtained. However it can be produced by placing the teeth directly on the reed to minimize damping. All except the very highest notes of the clarinet (those above about F_6) have their playing frequencies near an input impedance peak similar to the lower frequency intersection in Fig. 2.

In addition to the primary means of energy production which takes place at the fundamental frequency, it is also possible for energy to be added to the system at any of the harmonic components of the generated tone. We see from Eqs. (13c) and (13d) that the denominators of the expressions for the amplitude of the nth component p_n vanish under the same conditions which cause the denominators of the expressions for p_1 to vanish. Thus the conditions on z_n and $A(n\omega)$ to maximize the amplitude of the nth component are the same as the conditions on z_1 and $A(\omega)$ to maximize the energy production at the playing frequency ω. p_n is increased either by increasing z_n, which can be done by moving an input impedance peak nearer to a harmonic of the playing frequency, or by increasing $A(n\omega)$, which is done by moving the reed resonance frequency nearer to such a harmonic. In either case if $z_n A(n\omega) \simeq 1$, then additional energy is added to the system at the nth component, and the constraint of the additional feedback loop makes the regime much more stable in amplitude, frequency, and harmonic content. Incidental frequency modulation and spurious noise are generally reduced, and the attack and decay transients are shortened and stabilized. Of course it is quite unlikely that the denominators of all of the expressions of Eqs. (13b), (13c), and (13d) would rigorously vanish at the same ω for any value of n. The value of p_0 can be adjusted to "fine tune" the denominators of both expressions in Eq. (13b) to zero, but in general that same value of p_0 would not make the denominators of Eqs. (13c) and (13d) also vanish. All that is really required to stabilize a regime of oscillation is that the denominator of p_n be small when the denominator of p_1 vanishes. Equations (15a) and (15b) need only be approximate equalities.

This additional means of energy production at the reed frequency was not included in Benade's original definition of a regime of oscillation. To include this effect, the definition should be changed as follows:

A regime of oscillation is that state of the collective motion of a nonlinearly excited oscillatory system in which the nonlinearity of the excitation mechanism collaborates with a set of the modes of the entire system (including any possible modes of the excitation mechanism itself) to maintain a steady oscillation containing several harmonically related frequency components, each with its own definite amplitude and phase.

(The wording in this definition has intentionally been made general enough to include oscillations in systems other than just reed woodwinds.) While the high-frequency oscillation whose fundamental is near the reed frequency is not formally included in this definition, it is similar enough in musical quality that it will be called the "reed regime". The definition now includes all normal musical oscillations of reed instruments, although it does not include the so-called multiphonics, most of whose components are inharmonically related.

II. EXPERIMENTS

A. Determination of the range of the natural frequency of the reed

The player has considerable control over the natural frequency of the reed. By tightening and loosening his embouchure he can change the playing frequency of notes in the clarion register by ± 0.6% (± 10 cents) very easily. It can be shown that this corresponds to changing the natural frequency of the reed about ± 15% per cent (250 cents). To find the actual range over which the reed frequency can be adjusted, the apparatus shown in the block diagram of Fig. 3 was used. The reed frequency was approximately determined by measuring the frequency of the reed regime oscillation while playing with

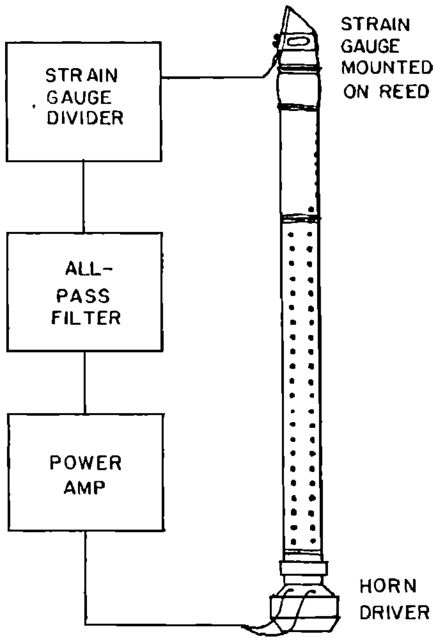

FIG. 3. Block diagram of apparatus used to measure reed frequency range under playing conditions.

a normal embouchure. Because the reed Q_r under normal playing conditions is too small to support a reed regime oscillation, the additional feedback through the electronic system was provided to allow the oscillation to be selfsustaining.

The operation of this additional feedback loop is as follows. A strain gauge mounted on the back of the reed is used as one leg in a voltage divider. As the reed undergoes its periodic motion, the voltage across the strain gauge acquires a small ac component proportional to the reed curvature at the gauge. This signal is amplified and fed to a driver at the lower end of the clarinet-like tube. Sound waves from the driver can cause the reed to vibrate thus providing the feedback necessary to set the system into self-sustained oscillation if the phase shift in the feedback loop is just right. The preferred oscillation frequencies are near that of the reed resonance and those of the standing wave resonances of the air column. The air column resonances can be heavily damped by placing glass wool in the tube, and the additional feedback is reduced at low frequencies by band-limiting the amplifier. Thus it is ensured that oscillation can only occur near the reed resonance frequency.

As mentioned before, with the amplifier turned off the system will not oscillate. If the amplifier amplitude response is flat and the phase response is tailored so that the phase shift in the feedback loop (including the shift due to the wave travel time up the tube) is a multiple of 2π radians at all frequencies, then turning up the amplifier gain would be equivalent to increasing the characteristic impedance of the tube. In both cases the pressure variation caused by a particular flow through the reed would be increased. As this input impedance is increased by increasing the amplifier gain, the system eventually goes into oscillation near the reed frequency where the impedance required for oscillation is minimum.

However, because in this type of oscillation the reed is driven near its resonance frequency, its response is nearly sinusoidal. Thus the feedback signal contains only a single frequency and the amplifier phase compensation can be considerably simplified. Since the phase shift needs to be accurate only at the oscillation frequency and not at its harmonics, a simple adjustable all-pass filter can be used to adjust the phase shift in the feedback loop. If the phase is adjusted so that the oscillation takes place with minimum gain, then the oscillation again takes place as if the characteristic impedance of the tube had been increased to the point of oscillation.

By playing with the electronics properly adjusted and with a range of embouchure settings essentially similar to those used in normal clarinet playing, it is possible to set the oscillation frequency anywhere between about 2 and 3 kHz. With somewhat more extreme changes of embouchure, the frequency can be lowered to about 1800 Hz and raised to about 3400 Hz. It can be shown that the reed resonance frequency will always be within about 10% of the playing frequency when playing in the reed regime. This reed frequency range seems reasonable in light of the results obtained in other experiments. Presumably the endpoints of the range would change somewhat for different reeds and different mouthpiece facing designs. Drastic deviation, however, would produce unacceptable playing behavior.

B. Experiments on a clarinet blowing machine

The theory of the preceding section predicts that second register oscillations should be stabilized if the reed frequency is set near a harmonic of the playing frequency. This section describes an experiment on a clarinet blowing machine which confirms that prediction.

The blowing machine used was that designed and built by Worman[2] in his earlier work. It consists of a rectangular cavity in which the mouthpiece is mounted. Clarinet-like upper joints can be attached to the outside of the cavity to create a normal musical air column. The dimensions of the cylindrical tube connecting the mouthpiece and upper joint are typical of clarinet barrels. The cavity surrounding the mouthpiece is connected through a length of $\frac{7}{8}$-in. copper tubing to the outlet of a reversed vacuum cleaner to provide the blowing pressure to the cavity. A brass "tooth", covered with a silicone rubber "lip", presses the reed against the mouthpiece facing simulating normal blowing conditions. The position of the "lip" and the force applied by the "tooth" can be varied with adjusting screws. The blowing pressure is adjusted by changing the line voltage of the vacuum cleaner with a Variac. After considerable practice it was possible to adjust the blowing pressure, "tooth" position, and "lip" force to set the system into oscillation at any note on the clarinet. The silicone rubber material used as the artificial lip provided significantly less damping than the human lip. Thus the reed Q_r in this experiment is higher than under actual playing conditions, and, in fact, is high enough to allow the reed regime to be produced.

The air column used was one specifically designed to have a very flat impedance beyond the cutoff frequency. Since this system does not have a speaker key or register hole like a normal clarinet, it was quite difficult to adjust the system to play in the upper register. The "tooth" position and "lip" force adjustments were critical. Both the low register note and the reed regime were much easier to obtain. However, when the adjustments were properly made the clarion register note could be played, and further delicate adjustments brought something approximating musical tone to the oscillation. With these adjustments made, a little damping material such as glass wool, or a handkerchief, was placed lightly just outside the first few open tone holes. This provided enough damping to lower the bore impedance peaks below the threshold for oscillation. The system would then jump to the reed regime and in all cases where the clarion register oscillation had been stable the reed regime frequency was within 3% (50 cents) of the second or third harmonic of the note in the clarion register which was played when the embouchure was set. For this experiment the clarion register regime was considered to be stable if the oscillation returned to the clarion register note when the damping was removed from the outside of the tone holes. The actual experimental results appear in Table I.

Thus, at least on the blowing machine and with an instrument whose impedance is flat beyond cutoff, in order to play with the best musical tone, the reed frequency should be set near a harmonic of the note being played and almost precisely at the frequency which maximizes energy production near the reed frequency. The few cents discrepancy between the harmonic of the clarion register note and the playing frequency of the reed regime is understood as follows: the clarion register plays at the frequency which maximizes the total energy production at both the playing frequency and the reed frequency, while the reed regime maximizes energy input at one frequency only. The additional constraint on the clarion register regime explains the observed small differences from exact harmonic relationship of the clarion regime and the reed regime.

C. Effects of reed resonance on spectrum and tone quality

It has been stated that if the reed frequency is placed just above a harmonic of the playing frequency, then that harmonic amplitude will be increased. It was also shown in the previous section that this setting of the reed frequency produces the best musical tone quality for clarion register notes on the clarinet. These two statements were reaffirmed in the following manner. A 3-mm-diam PZT ceramic microphone whose response is flat within about $\pm\frac{1}{2}$ dB over the range of frequencies considered, was mounted along the side wall of a mouthpiece to measure the mouthpiece pressure spectrum. By use of the experimental clarinet system with a flat impedance beyond cutoff, the mouthpiece pressure signal was recorded on a magnetic tape loop and played back through a GR 1900-A wave analyzer with its band-

TABLE I. Results of blowing machine experiment.

Clarion register note	Reed regime note	Component number matched	Cents deviation	Percentage frequency deviation
$D_4 - 38¢$	$A_7 - 20¢$	6	+18	+1.07
$B^\flat_6 + 20¢$	$F_7 + 65¢$	3	+45	+2.68
$C_6 + 30¢$	$C_7 + 32¢$	2	+2	·0.12
$C_6 + 8¢$	$G_7 + 4¢$	3	−4	−0.24
$C^\sharp_6 + 20¢$	$G^\sharp_7 + 40¢$	3	+20	+1.19
$D^\sharp_6 - 25¢$	$D^\sharp_7 - 10¢$	2	+15	+0.89
$E_6 + 10¢$	$E_7 + 0¢$	2	−10	−0.59

width set at 50 Hz. This is wide enough to span any small frequency shifts of any component of interest. Each note was recorded and analyzed three times—once at the frequency which gave the best tone quality from the musician's point of view, and once each at frequencies 0.6% (10 cents) sharp and 0.6% (10 cents) flat from that producing best quality. This corresponds to a shift of the reed's own natural frequency of about ±15% (±2-$\frac{1}{2}$ semitones). The spectra typically obtained from the recordings are of the sort shown in Fig. 4. These show that when the embouchure is adjusted for best tone quality the component at the reed frequency is maximized, and as the reed frequency is changed, sometimes the next component in the direction of the change is increased.

The results of these experiments under actual playing conditions can be explained as follows: One observes, as indicated in the last section, that the best musical quality occurs when the reed frequency matches a harmonic of the playing frequency. When this is true, the theory predicts that the component at the reed frequency is maximized. As the reed frequency is shifted, the amplitude of that component which had previously matched the reed frequency decreases drastically. If the reed frequency is moved far enough that it comes near to another component, then this harmonic amplitude increases for the same reason. Thus the results are consistent with the conclusions of the last section and agree with the predictions of the theory.

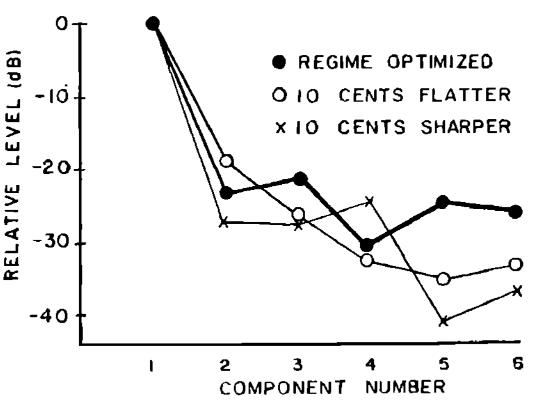

FIG. 4. Relative amplitude level of first six components of the played note G_5 played with best musical quality, as well as 10 cents sharp and 10 cents flat from this frequency. Reed frequency range spans the region of the third component.

IMPORTANT NOTE

One should *not* assume that these spectral changes alone are the reason for judging a note to have good musical tone quality. To the contrary, there are indications from practical music that the improvement is associated to a greater degree with the fact that when the reed frequency matches a harmonic of the playing frequency, the oscillation is stabilized by the increased feedback at the reed frequency. The incidental small frequency changes and the spurious noise present in the tone are thereby decreased. A proper study of such matters lies within the field of psychoacoustics rather than physics, and so lies outside the scope of the present inquiry.

D. Musicians' experiments

Since the recognition of the importance of reed resonance effects, several attempts have been made by Benade to use this information to understand better and to improve the playing quality of actual musical instruments. In all cases these experiments have given qualitative confirmation to the theory developed here. The first of these experiments involved placing the proper size blob of wax in the bell of a modern conservatory oboe. This rearranged the small impedance peaks and dips beyond the cutoff frequency in such a way that the playing qualities of some of the upper notes, which had previously been the worst on the instrument, were greatly improved. This occurred because the frequency of a small impedance peak beyond cutoff was made to be an integer multiple of the playing frequency, when the reed resonance frequency was also near such a multiple. However, for the oboe, the shape of the bell is important in determining the properties of the impedance below cutoff. Thus the tuning of several of the notes in the low register was affected in ways which would have required major surgery to correct. The method was not usable for this particular instrument.

On a different conservatory oboe whose impedance peaks are not well aligned at harmonic intervals, Benade has found that on most notes there are several distinct embouchures which achieve "best" musical tone for this instrument. By paying close attention to what he was doing with his embouchure, he has been able to find the origins of these "best-playing" embouchures. Care must be taken in analyzing such experiments, however, because changes in the embouchure change not only the reed frequency but also the effective volume of the reed cavity which for conical instruments will change the position and spacing of the impedance peaks. The different "best-playing" embouchures occur when various sets of air column impedance peaks are aligned at harmonic intervals with each other or with the reed frequency. These cases can be distinguished from each other in ways such as the following. When the embouchure is adjusted so that the playing frequency of the second register note is exactly an octave above the playing frequency of the low register note, then the first two impedance peaks are accurately aligned; this is one of the "best-playing" embouchures for the low register. Another occurs for the low register note when the reed frequency is at an exact harmonic of the second impedance peak. This embouchure can be identified because it is a "best-playing" embouchure for both the first and second registers. Many other examples of this type of behavior have been identified and explained. Of course an instrument with such multi-optimum behavior is not musically useful. The playing behavior of the entire instrument can be significantly improved, however, if for each note the impedance maxima are properly aligned with that embouchure which also placed the reed frequency at a harmonic of the playing frequency.

There is an exception to this general rule for the case of the Baroque oboe. This oboe has no register key and many octave changes are made simply by changing the embouchure. In order to accomplish this consistently and unambiguously, the embouchures for the two octaves must be different, and each must provide its own unique set of cooperations to stabilize the oscillation. In order to avoid unwanted octave shifts, the first and second impedance peaks should *not* be aligned with each other when the embouchure is set for either octave. This makes the reed frequency adjustment even more important because for many notes it is the only mechanism for additional energy input.

There are two aspects of clarinet behavior which can be explained using the ideas presented here. The upper register of the clarinet can be played without opening the register hole if the reed resonance is always properly adjusted at a multiple of the playing frequency. This is despite the fact that a low register oscillation can also occur and would be vastly favored were it not for the extra energy input to the upper register oscillation by the component near the reed frequency. As a second example, consider the very topmost notes on the clarinet. The notes above about G_6 (1400 Hz) are above the nominal cutoff frequency of the instrument and thus the impedance peaks at these frequencies are very small. It also turns out that the reed frequency cannot be comfortably lowered to place it at the desired playing frequency to produce a "reed regime." However, if the reed frequency is lowered sufficiently, then the resonantly enhanced reed transconductance can interact with the small impedance peak in the vicinity of cutoff to produce an oscillation which is a kind of hybrid of the normal and "reed regime" oscillations.

As a final example of the application of the ideas presented in this paper, many of the saxophone mouthpiece facing designs prevalent in the 1920's were such that the reed frequency could not be raised much above the playing frequency of notes in the top of the second register. The notes written at about D_6 could be achieved as reed regimes, but it was not possible to play many notes in the third register of the instrument. It was also not possible to play the second register without opening the register hole, because the reed frequency was too low to add energy to the oscillation at a higher component. More recent mouthpiece facing designs have allowed the reed frequency to be raised to a range analogous to that of the clarinet so that the third register is possible and the second register can be played without the register hole. The design of such facings can now be done as a conscious application of the phenomena discussed in this paper.

III. CONCLUSIONS

This paper has extended the work of Worman and Benade to show that the nonlinear flow control property of the reed, couples the reed resonance into the oscillatory energy production mechanism when the reed natural frequency is near to a low-order harmonic of the playing frequency. In this way the reed resonance can serve the same function as an input impedance peak in stabilizing an oscillation. The mathematics of the two cases is somewhat different since the reed resonance affects both the input impedance Z and the reed transconductance A, whereas an input impedance does not affect A. The musical significance of the two cases is, however, so similar that it has proven desirable to change the formal definition of a regime of oscillation to include those cases in which the additional energy input arises from a properly adjusted reed resonance.

Because of the analytical nature of Worman's method of solution, a number of simplifications have been made to the physical system to allow the major phenomena to be studied with a tractable mathematical formulation. However, a number of dynamically and musically important effects have been neglected which should be investigated in future studies. There are three major effects in this category. One is the Bernoulli force on the reed tip produced by the air flow through the reed. The Bernoulli force is very important in the double reeds and in single reed instruments whose mouthpiece design includes a high baffle at the mouthpiece tip.

The second is the large amplitude behavior when the reed beats against the tip of the mouthpiece. This again is especially important for double reeds which often beat even at fairly low oscillation amplitude.

The third phenomena yet to be investigated is the transient behavior of reed instruments. The present author has recently proposed a time domain method of solution which should be able to include all of these effects.[21] It is hoped that this solution can be implemented in the near future.

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