Physics of Music

Physics 341
Assignment 3
1)a) How many ocataves and semitones are the two notes with frequency 300 Hz and 2700 Hz apart?

Lets go up by octaves from the lowest note to the higher.

$$
\begin{align*}
2 x 300=6002 x 600 & =1200  \tag{1}\\
2 x 1200 & =2400  \tag{2}\\
2 x 2400 & =4800 \tag{3}
\end{align*}
$$

The last is too big. Thus to get to within one octave of the note we had to go up 3 octaves. The ratio between the final note and that final octave is

$$
\begin{equation*}
2700 / 2400=1.125 \tag{4}
\end{equation*}
$$

This is a tiny bit larger than one equal tempered tone 1.122. Ie the second note is 3 octaves and 1 tone higher than the first. (In fact it is exactly three octaves and one Pythagorian major second higher than the first).
b)A soprano sings two notes (in succession) a perfect fifth apart. What is the difference in frequency between the two notes if the lower one is sung at 470 Hz .

A perfect fifth is a frequency ration of $3 / 2$ (Pythagorian) or 1.4983 equal temperament. Thus the higher note will be $\frac{3}{2} 450=675 \mathrm{~Hz}$ or in equal temperament $1.4983 x 450=674.1 \mathrm{~Hz}$.
2)a) A workman is exposed to a sound of 60 dB for 7 hours, and 100 dB for one hour without hearing protection. What is the average energy rate of the sound that he received during the course of the day? The Workman's compensation says that if the average energy rate is higher that 90 dB during 8 hours, hearing protection must be provided. Is the company in compliance?

The second sound is 40 dB louder than the first which is 10000 times louder. Thus for 7 hours, the sound is a certain level, and the last hour it is 10000 times as much. On average it is therefor $(10000+7 \times 1) / 8$ times as loud as the reference, which was the 60 dB sound. This is on average $10000 / 8$ times as lound, or $40 \mathrm{~dB}-$ $9 \mathrm{~dB}=31 \mathrm{~dB}$ louder than the 60 dB . Ie the average is 91 dB which is 4 dB (about 2.5 times) higher than the limit. Remember that 10000 is 40 dB and $8=2 \times 2 \times 2$ or $3+3+3 \mathrm{~dB}$ (the difference between 10000 and 10007 is completely negligible as far as dB are concerned). Since we divide by 8 the 8 decreases the 10000 level so that is why we get $40 \mathrm{~dB}-9 \mathrm{~dB}$

We could also do this referencing everything to $0 \mathrm{~dB}\left(10^{-12} \mathrm{w} / \mathrm{m}^{2}\right)$ Thus 60 dB would be a million times as intense, or one millionth of watt per square meter. 100 dB would be 10 billion times louder, of one hundredth of a watt per square meter. Thus the average would be $7 \times$ amillionth $+1 \times$ ahundredth
divided by 8 hours total, or one 8 hundredths of a watt per meter squared. This would be $100 \mathrm{~dB}-9 \mathrm{~dB}=91 \mathrm{~dB}$.
b) The standard in BC for Railway workers is that the average noise level must not exceed 87 dB for an 8 hour day. BC mandates the 3 dB rule, namely that the time of exposure must be halved for each 3 dB rise in the average noise level. How long could a railway worker work in a place (eg a disco) with an average noise level of 120 dB ? In Ontario, the requirement is that the worker is allowed to be exposed to 90 dB for an 8 hour day, and that the time is halved for each 5 dB rise in noise level. How long would a Ontario worker be allowed to work in that same disco.

In BC , that 120 dB sound is 33 dB above the limit, which is 11 factors of 3 dB . Thus you need to divide 8 hours by two eleven times, which is very close to dividing by $2000\left(2^{1} 1=2048\right)$. Ie, the worker could be in that disco only ( $8 \times 60 \times 60$ ) $\mathrm{sec} / 2000=14.4$ seconds. $(15$ seconds)

In Ontario, that sound is just 30 dB above the limit (since the limit is higher), which is 6 times the 5 dB halving time. The limit would be $8 \mathrm{hr} /(2 \times 2 \times 2 \times 2 \times 2 \times 2)=1 / 8$ hr , or 7.5 min . which is about 30 times as long as the BC worker could be there.
4) When I sing in bed, after I stop it seems that the bed is singing a note back to me. What is going on here?

Most of the time when something makes a noise when you make a noise at it, it is because of resonance. (Now when you shout at your boy/girl frind and they shout back, one might be able to call that metaphorical resonance, but the physics is very different.) The notes from my mattress must mean that there is something which vibrates with a fairly high $Q$ within the mattress, and teh only possibility is the springs. Since my singing is at at least 100 Hz , the mattress kept singing for about 20 cycles, and thus had a Q of about 80. This very undamped which is what make the effect so surprizing.
4) This problem will be on the next assignment.
5) The sun at high noon in the tropics shines on the ground with an intensity of $1 \mathrm{~kW} / \mathrm{m}^{2}$. If this were sound instead of light, what would be the intensity of the sound in dB on the standard scale with the reference of $10^{-12} \mathrm{~W} / \mathrm{m}^{2}$ ?

The sun's is $1500 \mathrm{w} / \mathrm{m}^{2}$. To get to that starting at $10^{-12} \mathrm{w} / \mathrm{m}^{2}$ you have to multiply by 10 fourteen times and then by 15 . Each factor of 10 is 10 dB so after 14 of them, that is 140 dB . 15 is very close to 16 which is $2 \times 2 \times 2 \times 2-i 3+3+3+3$ $\mathrm{dB}=12 \mathrm{~dB}$. That gives a total of 152 dB . It would be a horrendously loud sound coming from the sun. Fortunately we see it rather than hear it. ( our eyes are much less sensitive than our ears to the energy coming in to them) If the whole sky were the same energy as the sun's surface as seen from earth, the level would be about the equivalent of 202 dB in sound energy, which is also the greater than level in a bomb blast (pressure variations of greater than 1 atmosphere pressure).

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