Pitch vs Frequency

There is a relationship between pitch and frequency. The faster the vibration of something producing a sound, the higher the pitch we tend to perceive that sound to be. This section will discuss this relationship.

Frequencies come in all values. between any two frequencies, no matter how close together they are, there is always some other frequency. However, for the purpose of music, this infinitude of possibilities it too great. Over the centuries, the set of all possible frequencies has been categorized into a finite set of frequencies which are used in the playing of music.

The first and most important aspect of pitches, is that there, given one pitch, there is another, either higher or lower, which sounds most pleasing, most harmonious when played together with the first. This "interval" (a distance between pitches) is called the octave. Two voices or two instruments played an octave apart, blend together into one sound. In fact so tight is the relationship, that one often says that even if two voices sing a song an octave apart, they are really singing the same note. There is, to some extent, a sort of equivalence between notes an octave apart, so much so, that one says that there is an "octave equivalence"– things an octave apart are really the same.

(Obviously this equivalence is limited in scope, because there are also clearly differences between two sounds an octave apart).

As was already discovered by Pythagoras, over 2000 years ago, there is also a very particular relationship between the frequencies of two notes an octave apart. The frequency of the higher note is exactly twice that of the lower note. No matter which note one plays, the note an octave higher also has a frequency of twice as much.

It is important to note that this relationship is somewhat strange. While musically, the interval of an octave is some set relationship between the two notes, the frequency difference between the two is not the same. It is the ratio of frequencies, not the difference, which is the same.

Of course, just playing notes an octave apart would be pretty boring. Thus, the musical

collection of useful frequencies must be larger than this. To this end, it has become traditional in European music to select a set of 12 pitches within every octave. Pieces of music tend to be restricted to sounding (sequentially or in parallel) a subset of these 12 notes. Usually, a piece of music does not encompass all twelve, but only a certain subset of these.

These 12 notes are supposed to divide the octave equally in some sense. Also the 12 subdivisions of one octave obey the octave relationship with those encompassing the next higher octave.

These subdivisions are called semi-tones. Since the octaves are characterized in frequencies by the ratio of the frequencies, so are the semi-tones. Ie, one note a semi tone higher than another has a frequency a set set multiple higher than the first. That ratio must be such that if one goes up by a semi-tone each time for 12 steps– ie such that the frequency is multiplied by that semi-tone ratio twelve times– one gets to an octave– ie the last one must have a frequency of twice the first..

This frequency ratio, which is such that if it is multiplied by itself 12 times to give 2, is called $2^{\frac{1}{12}}$. It has a numeric value of approximately 1.0595. In the so called equal tempered tuning, each semi-tone is exactly the same "size", ie, corresponds to exactly the same frequency ratio. Thus if we go up by 5 semi-tones, this corresponds to a frequency ratio in which the second note has a frequency of 1.059x1.059x1.059x1.059x1.059= $(2^{\frac{1}{12}})^5 = 2^{\frac{5}{12}}$=1.3348 higher than the first.

I. STANDARD

The above tells us how to go from one note to another a certain number of semi-tones higher (or lower) than another. But one needs some frequency to start from. The current standard is a frequency of 440 Hz, and that note is given the name A. This is the A440 standard. In the past, other frequencies have been used as the standard, so that for example in the baroque, the frequency of 415 Hz was used as the standard and called A.

The names for notes repeat in each octave. Thus the note one octave above A440 is also

called A, and would have a frequency of 880 Hz. The note an octave below A440 is called A, and has a frequency of 220 Hz.

Names are also given to other pitches between two successive A. However, instead of naming all 12 semi-tones, only a collection of 7 of the 12 pitches are named, using the letters from A to G.

The naming , starting from A440 as semi-tone number 0,

0–A

2–B

3–C

5–D

7–E

8–F

10–G

12–A

The unnamed notes in between are named using the #–meaning sharp, or "go up by one semi-tone", and "♭" or "Go down by one semi-tone". Thus the note between D and E for example could be called D#, which means go up by one semi-tone from D, or E♭, which means go down by one semi-tone from E.

The names are chosen such that as one goes from C to the C one octave higher, each of the named notes corresponds to one of the notes in the so called major scale starting on C. The major scale is a collection of notes, such that starting from the first note of the scale, called the tonic, one goes up by the sequence of

TTSTTTS

where T stands for a tone, which is two semi-tones, and S stands for semi-tone.

If one does not care which note the scale starts on but wants to indicate the various notes in the scale, one uses the nomenclature of

Do Re Mi Fa Sol La Ti Do

This is called SolFage, and is ancient. Guido d'Arrezzo talks about it around 1000AD.

In his system, Do as called Ut, and the syllables Ut Re Mi Fa Sol La were the first syllables in the lines of a very well known Latin song. The standard note to start his "hexachord" (6 note scale) on was G, or Gamma, thus giving us the word gamut, for Gamma-Ut, the hexachord in which the whole scale starts on the note Gamma.

This SolFage is still used by singers, and was made popular in the movie *The Sound of Music*.

The Do represents the starting note, the tonic, whatever that happens to be. Then Re is the note one tone, or two semi-tones, above the Do. Mi is one tone above Re, or two tones (4 semi-tones) above Do. Fa is one semi-tone above Mi. Sol is one tone above Fa, La a tone above Sol, Ti a tone above La, and the final Do a semi-tone above Ti.

In the scale of C (ie starting on C), Do would be C, Re would be D, Fa would be E, Sol would be F, La would be G, Ti would be A, and Fa would be B. Note that if one starts on C, all of the named notes fall into the scale, and none of the in-between notes are in the scale.

One could start the scale on another note. For example, starting it on A, we would have Do is A, Re is B, Mi (a tone above Re) would have to be the semi-tone between C and D. In order that each of the Do-Re-Me use a different letter name, we use the name C# to designate this note. Thus Mi would be C#.

Fa, a semi-tone above Mi, would be D, Sol would be E, La would be F#, Ti would be G#, and Do would be A again.

Ie, which element of the scale depends on where you start the scale.

One additional note. Since each octave above or below 440 Hz is called A, one sometimes wants a nomenclature for saying which octave one is talking about. This is often done with subscripts. The note A440 is called A_4. The note an octave lower is called A_3, and octave lower still (ie A110) is A_2, etc. The note an octave above A_4 is A_5.

However, just to confuse students, the turnover point in labeling subscripts is not A, but C. Thus A_4 is followed by B_4 but the next note is C_5, and not C_4. Similarly, the note just before A_4 is G_4, not G_3.

C_4 (about 261 Hz) is the so called middle-C on the piano.

II. HOW TO CONVERT PITCH TO FREQUENCY (A440 STANDARD)

This brief note gives the procedure for converting from pitch to frequency, with the standard that A is 440 Hz.

Pitch to frequency:

Determine which octave the pitch note is in. Count how many octaves the note would have to be moved to get into the "4th" octave. (ie so the note lies between C4 and B4).

Look up the frequency which corresponds to the note name in the table below

$$C4 (middleC) = 261.6Hz$$

$$C4\# = D4\flat = 277.2Hz$$

$$D4 = 293.7Hz$$

$$D4\# = E4\flat = 311.1Hz$$

$$E4 = 329.6Hz$$

$$F4 = 349.2Hz$$

$$F4\# = G4\flat = 370.0Hz$$

$$G4 = 392.0Hz$$

$$G4\# = A4\flat = 415.3Hz$$

$$A4 = 440.0Hz$$

$$A4\# = B4\flat = 466.2Hz$$

$$B4 = 493.9Hz$$

$$C5 = 523.3Hz$$

Now multiply the frequency by two for each octave above the 4th octave the note lies. (eg, G6 would have a frequency $2 \times 2 \times 392 = 1568.0Hz$) and divide by two for each octave below the 4th the note lies (eg, G3 would be $392.0/2 = 196.0Hz$.)

In th opposite direction, to find which note a certain frequency corresponds to , first divide or multiply successively by 2 until the frequency lies between 126.2 Hz and 523.3 Hz. Then look up the nearest result in the above table. Use the number of times you divided or multiplied by two to see in which octave that note name should go. Ie, all one to the 4 for each time you had to divide by 2, and subtract 1 from 4 for each time you had to multiply by two.

For example, 15700 Hz, which is the frequency of the fly-back transformer in a TV set.

$$15700/2 = 7850$$

$$7850/2 = 3925$$

$$3925/2 = 1962.5$$

$$1962.5/2 = 981.3$$

$$981.3/2 = 490.7$$

Now, 490.7 lies between 261.6 and 523.3. It lies closest to B4. Since we divided by two five times, the note would be approximately B9.

III. GRAPHING

It is often more convenient to graph against pitch rather than against frequency. However at the same time, it is often useful to put the frequencies rather than the pitches onto the axis of the graph. This means that the frequencies are plotted in a very uneven fashion on the graph. On the next page, I have plotted one of these. On the bottom axis of the graph I have places the frequencies, with major vertical lines occurring at 100 Hz, 1000 Hz and 10000 Hz. On the top of the graph I have plotted the pitches, placing a major tick mark at each A, and a minor tick mark at each full tone (not half tone).

IV. INTERVALS AND FREQUENCY RATIOS.

The musical intervals can be expressed in terms of frequency ratios. If one tone is say a (perfect) fifth above another, its frequency is a factor of 1.4983. Here is a list of the semi-tones their interval name, and the frequency ratio they correspond to

$Semi-tones$	$Interval$	$Ratio$
1	$(Minor Second)$	1.059463094
2	$(Major)$**Second**	1.122462048
3	$(Minor Third)$	1.189207115
4	$(Major)$**Third**	1.259921050
5	$(Perfect)$**Fourth**	1.334839854
6	$(Augmented)Fourth$	1.414213562
7	$(Perfect)$**Fifth**	1.498307077
8	$(Augmented)Fifth$	1.587401052
9	$(Major)$**Sixth**	1.681792831
10	$(Minor Seventh)$	1.781797436
11	$(Major)$**Seventh**	1.887748625
12	**Octave**	2.

$$\tag{1}$$

We will use only the names corresponding to the unbracketed terms.

To use this, let us say we want the note a fifth above A440. The name of the note will be E. 7 semi-tones above A4 will be E5 natural. The frequency would be $1.498 \times 440 = 659.1 Hz$. (remember that the octave number changes at C)

The note a twelfth above A, is an octave (an eighth) plus a fifth above A4 which is a fifth above A5 which is E5. (Frequency an octave above E5 =1319.2Hz)