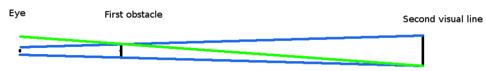

Figure 1:

Atoms to Universe
Physics 340
Assignment 1

1. How for away must I stand from you so that my fist (diameter about 10 cm) just covers the rising full moon?

The angle subtended by object is given (in radians) by the ratio of the size over the distance of the object. To get degrees, you mutiply radians by 57. The moon is 1/2 a degree (just over 30 minutes of arc) in size, which is (1/2 degrees)/(57 degrees per radian) = 1/114 radians. My fist is 10cm in diameter, so to get how far it would have to be to cover 1/2 a degree, would be 114 times as far away as its size. which would be 1140 cm which is about 11 meters away.

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2. Draw a diagram. On the left side of the page draw a dot. 5 cm horizontally from the dot draw a vertical line 0.5 cm high so the centre of the line is on the horizontal line through the dot. 20 cm from the dot draw a vertical line 1.5 cm long centred on the same horizontal. The dot is your eye. The second vertical line is hidden behind the first. How far do you have to move your eye vertically so that is just starts to see one edge of the second line? How far do you have to move your eye vertically so it can see the whole of the second line? (You may do this with the diagram, and with measurement rather than with arithmetic)

See the diagram. Along the blue line, one would just see the edge of the far line past the closer vertical line. This would represent about a .15 cm motion of the eye from the centerline. If the eye were along the green line, you could see the whole of the far vertical line past the nearer one. This, along the vertical line through the eye is about .7 cm above the midpoint location of the eye. Note that this is larger than than the nearer vertical line, while the former was much smaller.

This is supposed to crudely represent a solar eclipse where the second line represents the diameter of the sun, and the first line is the diameter of the moon. If the distance you move your eye is half the width of the full eclipse shadow. is that distance half the length of the first line? Anaxagoras deduced that the diameter of the moon equalled the width of the eclipse shadow. From this exercise does that make sense? (The full eclipse shadow is the region where the moon completely hides the sun).

(Note that the proportions in this exercise are not correct for the moon and sun. The distance from us to the moon is about 100 times the moon's diameter, which would be really hard to draw.)

3. Anaxagoras thought that the diameter of the moon was about the same as the Peloponnese because that was the was the area covered by the total eclipse. The Peloponnese is about 100 km across. How far away would the moon be in that case? If one travelled from Athens to Macedonia (about 600km) by how much would the location of the moon in the sky change due to parallax?

The moon's angular size is about 1/114 radian as in problem 1. It is 100km across so it will be 114 times further away or about 11000 km away. That about the diameter of the earth (although he did not know what the diameter of the earth was). But Macedonia, which the Greeks certainly knew about, is about 600km away. The parallax is the distance travelled over the distance to the object, or 600/11000 which is about 1/20 radians, which is about 3 degrees. Ie, over this short journey the moon would move with respect to stars by about 3 degrees. This would be highly visible.

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4. If the earth (diameter about 6400 km) rotates, and the moon is 60 earth diameters away (Hipparchus's figure), what would the change in parallax of the moon be between the beginning to the end of the night? How far would the moon have travelled through its orbit during that time? (take a lunar month as 28 days)

[This problem was badly set up since the diameter of the earth is about 12800km. What I gave was the radius of the earth. I will use the true diameter below, but by dividing by half you can get the answer to the figures I mistakenly gave above]

During the night, the observer would travel about the diameter of the earth, or about 12800. The moon is 60 times the diameter away, so the paralax of the moon during that travel would be 1/60 radians, or about 1 degree. Over 24 hours the moon travels 1/29 of a circle (29 days in a month) , so in 12 hours it travels (360/29)/2= 3 degrees. The parallax is about 1/3 of the angle the moon travels and would be easily measured and visible.

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5. Eratosthenes was the first to measure the size of the earth. Tell me how he did it. What kinds of error could have crept into his result? (You may use

whatever sources you wish, but try to make sure that they are reputable, and tell me what they are.) What is the circumference of the earth? This figure was well known in 15th century Italy. It was also known that the overland route to China was about 10000km, from traders along the silk Route. Thus how far would Columbus have had to sail to get to China if the Americas were not in the way? At a speed of about 4 knots (about 7km/hr) how long would the Journey have taken?

One of the key errors would have been in he distance between Alexandria and Syene. One way would be to count foot strides betwen the two. The problem is that even a trained walker could not do better that about 10 percent regularity in the number of strides. A better way would have been to use the circumfernce of a wagon wheel, and cound how many turns of the wagon wheel would get one between the two locations. This could well give about 1accuracy. (Probably the main problem would be keeping proper count of how many turns).

Another problem is that Syene is not directly south of Alexandria. Thus the walk along the diagonal would add a small amount (probably less than 5% to the distance)

The second problem would be in the measurement of angles. The well, being a deep well, one could hang a plumb-bob down to figure out how far from the vertical it was. Similarly the pole at alexandria could be made accurately vertical with the same plumb-bob (a weight hung from a string to accurately measure the vertical). In both cases the angle could be measured to probably 1/10 of a degree. This would give about a 2 or 3 % error in the determinationof the angle difference between Alexandria and Syeme of the sun. So the fact that his measurement was as good as it was was probably not an accident, and seems reasonable.

One of Columbus' "contributions" to science was to insist that the earth was less than half of Eratosthenes' figure. He was lucky!

[Brief table of commonly used prefixes: n = nano = 10^{-9} = 1/1,000,000,000
μ = micro = 10^{-6} = 1/1,000,000
m = milli = 10^{-3} = 1/1,000
c = centi = 10^{-2} = 1/100
d = deci = 10^{-1} = 1/10
h = hecta= 10^{2} = 100
K = kilo = 10^{3} = 1000
M = Mega = 10^{6} = 1,000,000
G = giga = 10^{9} = 1,000,000,000]

It is interesting that in scientific notation, names are given only up to Y= Yotta= 10^{24}, whereas in classical Japanese there are names for numbers at least all the way up to 10^{52}.

http://en.wikipedia.org/wiki/Japanese_numerals.

(The Japanese use $10000 = 10^4$ as the multiple for names, rather than our 1000.) Why in the 16th century anyone would need to give such a large number a name I do not know. This aside is of course totally irrelevant to the course.