LUNAR ECLIPSE TIMES PREDICTED BY THE BABYLONIANS

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1. Introduction

Babylonian astronomers recorded many observations of lunar and solar eclipses. These are preserved in the Late Babylonian texts recovered from the ruins of Babylon during the last century, and now largely held in the British Museum. The texts, most of which were badly damaged when they were first dug up, contain details of lunar eclipses from around 750 B.C. to 50 B.C. and solar eclipses from 350 B.C. to 50 B.C. The texts also contain a number of predictions of both lunar and solar eclipses that did not prove visible at Babylon. It is not known exactly how the Babylonians made their eclipse predictions, but it seems clear that they knew of a number of cycles that gave rise to the times of eclipse possibilities. These cycles were based upon observations stretching back many years. The most successful eclipse cycle is the Saros, an eighteen-year cycle,¹ and it seems that the Babylonians were using this by at least the middle of the sixth century B.C.²

Comparison of the time of predicted eclipses with computation provides a method for determining the meaning of the terminology used, and a measure of the precision to which the Babylonian astronomers could predict the time of an eclipse. In this paper, we concentrate our attention specifically on lunar eclipse predictions, since direct comparison between prediction and modern computations is possible. Calculations of the local time of occurrence of a solar eclipse are greatly affected by geographical circumstances and so the interpretation of timing errors is more complex.

In order to compute accurately the times of eclipses in the past it is necessary to take into account changes in the rate of rotation of the Earth due to tides and other causes. These changes, which have been extensively investigated by Stephenson and Morrison,³ lead to significant clock errors. Before commencing our study we shall collate observational data from the Late Babylonian period and produce a standardized curve of the clock error that we shall use in calculating the times of the predicted lunar eclipses.

In this paper we shall analyse all the extant lunar eclipse predictions made by the Babylonians for which a timing is fully recorded, to determine what is meant by the terminology used in the prediction, and to provide a measure of the accuracy of the predicted times.

2. Sources of Data

There are four main types of Late Babylonian astronomical text which contain information on lunar eclipses: astronomical diaries, 'goal-year texts', eclipse lists, and individual eclipse reports. The diaries contain observations of celestial phenomena made and recorded on a daily basis by the astronomers. The earliest existing diary has been dated to 652 B.C., and survivals occur with generally increasing frequency down to the mid-first century B.C.⁴ From around the third century B.C. onwards (at least from 236 B.C. to 24 B.C.), the astronomers assembled past observations from the diaries and recorded them on goal-year texts; each goal-year text contains reports of lunar eclipses from 18 years previously and selected planetary data to assist in the making of predictions. Around the same period, eclipse tables were also produced from the diaries; the surviving tables contain lists of eclipses stretching from about 750 B.C. to 160 B.C. Individual eclipse reports date from 170 B.C. to 66 B.C.

The first detailed classification of the Late Babylonian astronomical texts (hereafter: LBAT) was attempted by Abraham Sachs in 1948.⁵ Sachs later worked extensively on translating the astronomical diaries. This was continued after his death in 1983 by Hermann Hunger, who has published transliterations and translations of virtually all the dateable diaries.⁶ In an unpublished, but freely circulated, manuscript,⁷ Peter Huber has translated virtually all of the records of lunar and solar eclipses recorded on the surviving goal-year texts and eclipse tables, and abstracts from copies of the diaries that were available to him.

Observations of eclipses by the Babylonian astronomers usually contain the time of the eclipse measured relative to sunset or sunrise, the approximate entrance angle of the shadow, and an estimate of the eclipse magnitude. In many reports the duration of the individual phases of the eclipse are also given. The Babylonian unit of time was the *uš*, which equals 4 minutes.⁸ The *uš* corresponds to the time for the Earth to rotate through one degree, and so is customarily rendered as 'degree' in translations.

An example of a lunar eclipse record is:

B.C. 397 Apr. 5.

"Month XII_2 14. Beginning on the south side, a quarter of the disk covered. It became bright toward the west. 27 degrees total duration.... At 48 degrees after sunset." [LBAT 1416 Rev. II middle; transl. Huber, p. 41.]

The record states that the eclipse started from the south (bottom) of the Moon, and at mid-eclipse the Moon was a quarter covered by shadow. The eclipse cleared from the west (right), and lasted for 27 degrees (1.8 hours). It is characteristic for the record of the observation to end with a time. This may be understood as referring to the time of first contact. Direct evidence for this inference can be found in a small number of texts. In the following example, the record of the observation allows the first contact time to be deduced directly:

B.C. 353 Nov. 21.

"[Artaxeres III, year 6], month VIII 14, beginning on the south-east side. After 23 degrees, total. 18 degrees duration of maximal phase. After 6 degrees of night, a quarter of the disk had become bright and it set eclipsed ... at 47 degrees before sunrise." [LBAT 1414, Rev. III, bottom, 1–9; transl. Huber, pp. 49–50.]

In this example, the Moon was totally eclipsed. It took 23 degrees for the eclipse to become total, and totality lasted for 18 degrees. 6 degrees after the end of totality, the Moon set. Clearly first contact is 23 + 18 + 6 = 47 degrees before the Moon set, as stated at the end of the record. Hence it may be inferred that the statement "... at 47 degrees before sunrise" indeed relates to the time of first contact.

Observations of a total solar eclipse, during which four planets and several stars were visible at Babylon, are recorded on two separate tablets: a goal-year text (LBAT 1285) and an astronomical diary (BM 45745).

B.C. 136 Apr. 15

"Month XII₂ 29. Solar eclipse, begining on the south-west side. In 18 degrees of day ... it became total. At 24 degrees after sunrise." [LBAT 1285; transl. Huber, pp. 93–94.]

"... day 29. At 24 degrees after sunrise, solar eclipse; when it began on the south and west side ... Venus, Mercury, and the Normal Stars were visible; Jupiter and Mars, which were in their periods of invisibility, were visible in its eclipse.... It threw off (the shadow) from west and south to north and east; 35 degrees onset, maximal phase, and clearing." [BM 45745; transl. Sachs and Hunger.]

The astronomical diary clearly implies that the eclipse started at "24 degrees after sunrise", the same time as given at the end of the observation in the goal-year text. From this evidence we may safely conclude that the time stated at the end of an observational record of a solar or lunar eclipse is the time of first contact.

The eclipse records also contain descriptions of lunar eclipses that proved invisible at Babylon. These descriptions must then be predictions rather than observations. Usually it is possible to distinguish a predicted eclipse from an observation by the terminology used. Ordinarily $an - ge_6 sin$ is used for a prediction, whereas the opposite order $sin an - ge_6$ is an observation. An eclipse which was not expected to prove visible is indicated by $sa \ dib$, meaning "which passed". Predicted eclipses which were expected to be visible but did not prove so are indicated by $ki \ pap \ nu$ igi, "Observed, but not seen".⁹ Perhaps a better rendering of the term translated by Huber as "observed" would be "watched for".

Descriptions of eclipse predictions are usually brief, giving only a single time. For example:

B.C. 194 May 11

"Year 118, month II 13, which passed. At 94 degrees [after sun]rise." [LBAT 1436 Obv. 2; transl. Huber, pp. 58.]

B.C. 170 Aug. 13

"Month V 15, lunar eclipse.... Observed, but not seen. At 4 degrees before sunrise." [LBAT 1263 Rev. 8'; transl. Huber, pp. 62–63.]

Comparison with the preceeding examples of observational records suggests that the eclipses were expected to begin at 94 degrees after sunrise and 4 degrees before sunrise respectively. In an example from 67 B.C. we have a mixture of a prediction and an observation:

B.C. 67 Jan. 19

"Year 180 Month X 15. Moonrise to sunset: 1 degree. As the Moon rose, twothirds of the disk on the north-east side were eclipsed. 6 degrees of night duration of maximal phase.... In 16 degrees of night, from the south-east to northwest it became bright. 23 degrees total duration.... At 16 degrees before sunset." [LBAT 1448; transl. Huber, pp. 77–78.]

In this example, the Moon rose eclipsed. The end of the record gives a time of first contact before the Moon rose which clearly must be a calculation. It is not clear whether this time was predicted in advance, following the usual procedure, or was calculated retrospectively, based upon the measured times of the subsequent phases.

There are 35 lunar eclipse predictions for which a time is recorded in the diaries, goal-year texts and eclipse tables available to us. Of these, 27 predictions describe eclipses "which passed". The eclipse of 170 B.C. is said to have been "Observed, but not seen". The other 7 eclipse records are damaged, but give a time during the hours of daylight when the Moon was below the horizon and could not have been seen. Although we infer that the predicted times relate to first contact, we shall leave this option open at this stage. In addition to the above examples, there are 8 records of observed eclipses that give a time of first contact before the Moon rose.

3. Variations in the Rate of Rotation of the Earth

When modern calculations of eclipses are made, they yield the times of the various phases of an eclipse in Terrestial Time (TT), which is defined by the motion of the Moon and planets. Babylonian observations of eclipses are expressed relative to the time of sunrise or sunset for the observer. Local times (LT) can easily be converted into Universal Time (UT), defined by the rotation of the Earth, by adjusting for the equation of time¹⁰ and the geographical longitude. Due to the action of tides and other factors, the rate of rotation of the Earth is not constant, and so neither is the length of the mean solar day. During the Late Babylonian period, the mean solar day was only about 0.05 seconds shorter than today, but the accumulated clock error in the million or so days that have elapsed since then amounts to several hours. This clock error is equal to the difference between Terrestrial Time and Universal Time (in the sense TT–UT), and is known as ΔT .

If the change in the rate of rotation of the Earth was due only to the tides, then ΔT would be represented by a closely parabolic equation, but this however is not the case.¹¹ Other causes of variation in the length of the day probably include postglacial uplift and changes in sea-level. The adopted values of ΔT for the Babylonian era have been determined from a series of observations of both lunar and solar eclipses made by the Babylonian astronomers themselves. For all timed contacts, we have determined the LT by first calculating the time of sunrise or sunset, and then converting to UT. The TT of the contact was computed separately and the difference between the TT and the UT gave the value of Δ T in each case. A lunar acceleration of $-26''/cy^2$, as first derived by Morrison and Ward¹² and recently confirmed by Dickey *et al.*,¹³ was adopted in all our computations.

Due to the inaccuracies of timing methods used by the Babylonians only timed intervals of less than 25 degrees (that is, less than 100 minutes) were considered, as these individual ΔT results show a remarkably small scatter. For significantly longer time-intervals the scatter is much larger; evidently the clepsydras used by the Babylonians were subject to considerable drifts over several hours. Table 1 shows the value of ΔT calculated for each of the selected eclipses.

The total solar eclipse of 15 April 136 B.C., during which four planets and several stars were seen at Babylon, provides critical limits to ΔT on that date. The mere occurrence of totality in Babylon independently and accurately confines ΔT to the range 11210–12140 seconds at that date, without any need for a measurement of time (although as noted above the times of the individual phases were in fact measured).

Figure 1 shows a plot of the values of ΔT in the Late Babylonian era, based upon the data in Table 1 and the single total solar eclipse. Each value of ΔT determined from a timed eclipse measurement is plotted as a point, and the range determined by the total solar eclipse of 136 B.C. is shown by the vertical line. Over this relatively short period of time (a few centuries), the ΔT plot can be approximated by a straight line; fitting a more complicated function might be misleading. Clearly, for longer timescales a more detailed treatment would be needed. The equation of the line (shown as the dashed line in Figure 1) is:

$$\Delta T = 9520 - 1612t \text{ sec}, \tag{1}$$

where t is in centuries from the year 1 B.C. (year 0 on the astronomical scale). The value of ΔT determined by the eclipse of 215 B.C. is clearly too high relative to the remaining very self-consistent results; it is thus not included in calculating the line of best fit. Presumably the text containing this eclipse contains a scribal error. There is a close accord between the range of ΔT obtained from the total solar eclipse of 136 B.C. and the individual results from timed observations, an encouraging situation.

4. Analysis of Eclipse Predictions

Initially confining ourselves to the records purely containing predictions, in Table 2 we list the 35 eclipse predictions for which a time is recorded. We were able to identify 19 as corresponding to umbral eclipses and 12 as penumbral eclipses. The other 4 predictions do not relate to real eclipses (on the stated dates, the Moon passed completely to the north or south of the Earth's shadow). The late Babylonian astronomical texts do not contain any *observations* of penumbral eclipses, and so we feel that only predictions relating to umbral eclipses can be regarded as being "successful". Predictions relating to penumbral eclipses must be regarded as having been "near misses". We note that the Babylonian astronomers were successful in their predictions 55% of the time, an additional 35% being near misses.

TABLE 1. ΔT results determined from Late Babylonian lunar and solar eclipse timings estimated to th	e
nearest degree (interval measured less than 25 degrees).	•

Julian Year.	ΔT (sec.)	Julian Year.	ΔT (sec.)
-665	21050	-239	14200
-536	18800	-214	17750
-482	17300	-211	11800
-420	15500	-193	13650
-407	15250	-188	11250
-406	16200	-169	12300
-405	16500	-169	12200
-352	14550	-142	12550
-352	15250	-133	10950
-321	14100	-128	12600
-316	15550	-108	12050
-307	14100	-66	10150
-280	12950		

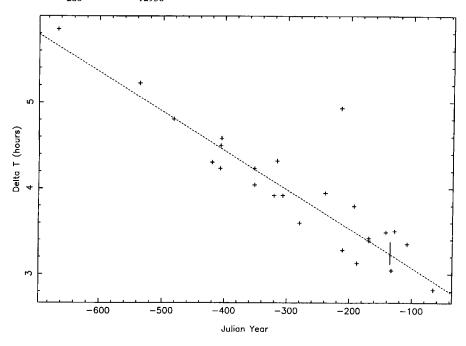


FIG. 1. ΔT plot for the Late Babylonian era of all timed eclipse data measured to the nearest degree (interval measured less than 25 degrees), and the total solar eclipse of 136 B.C.

First we will consider the successful predictions. For each prediction we have calculated the time of sunset or sunrise to yield the LT of prediction. Using modern ephemerides, we began by calculating the TT of first and last contact for eclipses occurring on the date of each prediction. Using Equation (1) to give the value of ΔT for the year of the eclipse, we then deduced the UT. By adjusting for the equation of time and the geographical longitude, we obtained the LT at Babylon of first and last

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Julian Date	Description	Predicted LT (hours)	Category
-730 Apr 9	Passed 60 after sunrise	9.74	Successful
-667 May 2	Passed 40 after sunrise	8.03	Successful
-649 May 13	Passed 60 before sunset	14.80	Successful
-572 Sep 25	Passed 35 before sunset	15.80	Successful
-566 Feb 25	3 before sunrise	6.22	Failure
-439 Jul 28	Passed with sunrise	4.99	Successful
-409 Jun 28	70 after sunrise	9.50	Successful
-396 Sep 30	80 before sunrise	0.63	Near miss
-388 Oct 31	Passed 60 after sunrise	10.48	Successful
-379 Oct 22	20 after sunrise	7.66	Failure
-356 Feb 14	Passed 40 before sunset	14.77	Near miss
-352 May 28	7 after sunrise	5.47	Successful
-334 Dec 3	Passed 60 before sunset	13.09	Successful
-291 Aug 11	Passed 27 after sunrise	6.97	Near miss
-278 Jun 19	18 before sunset	17.95	Successful
-278 Nov 13	Passed 45 after sunset	20.31	Failure
-248 Apr 18	Passed 39 before sunset	15.91	Near miss
-248 Oct 13	Passed 30 after sunset	19.78	Near miss
-232 Dec 14	Passed 74 after sunrise	11.91	Successful
-225 Feb 6	Passed 30 before sunset	15.34	Successful
-194 Jun 20	Passed 15 before sunset	18.16	Near miss
-194 Nov 16	Passed 45 after sunset	20.28	Near miss
-193 May 11	Passed 94 after sunrise	11.46	Successful
-184 May 30	15 before sunset	18.04	Failure
-172 Mar 21	Passed 47 after sunrise	9.10	Near miss
-169 Feb 16	Passed 31 before sunset	15.42	Successful
-169 Aug 13	Not seen 4 before sunrise	4.94	Successful
–168 Jan 7	Passed 7 before sunrise	6.49	Near miss
-162 Sep 23	Passed 48 after sunrise	9.07	Successful
-161 Feb 18	Passed 31 before sunset	15.45	Near miss
-161 Aug 14	Passed 25 before sunset	17.10	Near miss
-140 Jul 22	Passed 34 before sunset	16.76	Successful
-136 Oct 5	Passed 79 before sunrise	0.81	Near miss
-86 Aug 24	Passed 30 after sunrise	7.38	Successful
-76 Feb 9	Passed 76 after sunrise	11.68	Successful

TABLE 2. Lunar eclipses predicted by the Late Babylonians for which a time is fully recorded.

contact. Subtracting from our calculated LT the LT of the prediction, we obtained the error in the predicted time. This error — on the alternative assumptions that the predicted time is that of either first or last contact — is shown in Table 3. The mean error in first and last contact has been determined as -0.02 hours and +2.63 hours respectively. The difference is the mean duration of the lunar eclipses in the sample, and so it is apparent that the times given in the predictions are those of first contact. Figures 2(a) and 2(b) show the errors in the prediction assuming (a) first contact, and (b) last contact. The dotted lines show the mean error in each case.

It is clear from the records of eclipse observations that the Babylonian astronomers identified the various phases of an eclipse, and recorded their durations together with the time of first contact. Using a cycle to predict eclipses based upon past observations, the first contact time would be the easiest to obtain, and would give the astronomers a time to watch to try and confirm the prediction. Furthermore,

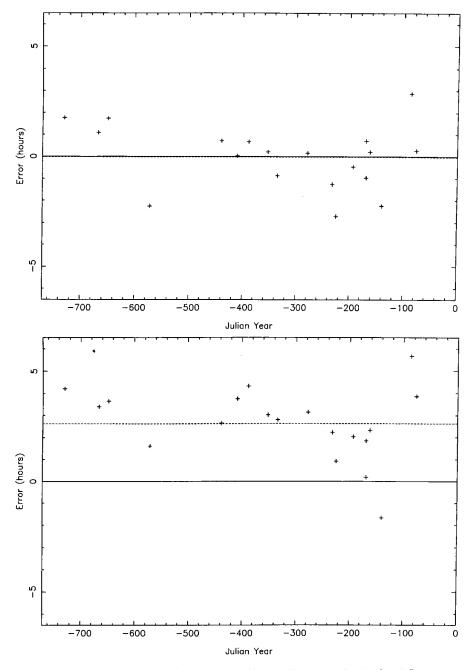
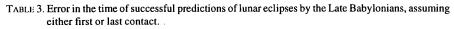


FIG. 2. Errors in the time of successful predictions of lunar eclipses assuming (a, above) first contact and (b, below) last contact.

Julian Year	First Contact Error (hours)	Last Contact Error (hours)
-730	+1.77	+4.21
-667	+1.09	+3.39
-649	+1.74	+3.64
-572	-2.25	+1.61
-439	+0.71	+2.65
-409	+0.03	+3.75
-388	+0.67	+4.33
-352	+0.21	+3.02
-334	-0.87	+2.80
-278	+0.15	+3.14
-232	-1.26	+2.23
-225	-2.72	+0.91
-193	-0.47	+2.03
-169a	-0.96	+0.18
-169b	+0.70	+1.84
-162	+0.20	+2.32
-140	-2.25	-1.67
-86	+2.86	+5.66
-76	+0.26	+3.86



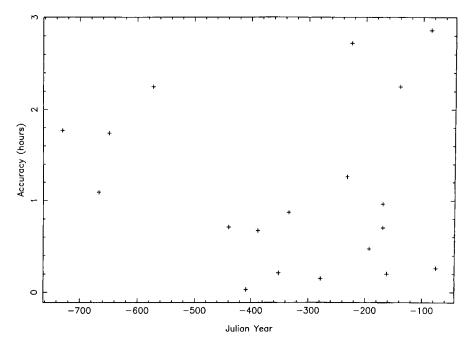


FIG. 3. The accuracy of successful predictions of lunar eclipses by the Late Babylonian astronomers.

TABLE 4. Error in the time of near miss predictions of lunar eclipses by the Late Babylonians.

Error (hours)
+1.13
-0.13
+0.60
6.80
-2.26
+0.86
-3.66
+1.53
-6.46
-7.26
-0.13
+2.06

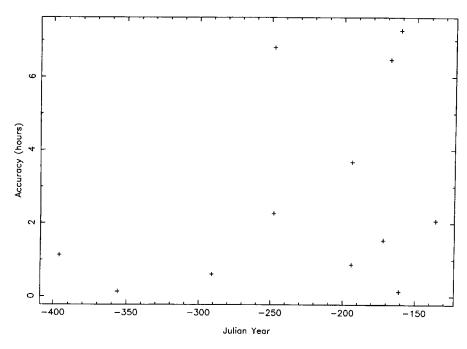


FIG. 4. The accuracy of near miss predictions of lunar eclipses by the Late Babylonian astronomers.

if the predictions were used for religious purposes, it seems likely that they would wish to know the time when the eclipse would begin.

We define the accuracy of an eclipse prediction as the error between the predicted and computed times, irrespective of whether the prediction is early or late. This is thus the modulus of the error of prediction. Figure 3 shows the accuracy of the predictions in our sample on the assumption that the time given is first contact. The mean accuracy of prediction is 1.12 hours or 17 degrees. Before about 550 B.C. the Babylonian astronomers quoted the times of observed eclipses to the nearest 5

	Julia	in Date	Description	Predicted	LT (hours)	Error (ho	urs)		
	-590	Mar 22	30 before sunset	t 15	.98	+0.06			
	-562	Sep 5	35 before sunset	t 16	.13	+0.76			
	-189	Aug 23	30 before sunset	t 16	.63	-0.94			
		Aug 23	42 before sunset	t 15	.83	-0.11			
		Sep 24	30 before sunset		.09	+0.77			
		Jan 19	16 before sunset		.07	-0.81			
		Jan 8	30 before sunset		.04	-0.50			
	-65	Dec 28	6 before sunset	t 16	6.60	-0.03			
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TABLE 5. Error in the time of first contact when the Moon rose eclipsed.

FIG. 5. The accuracy, for lunar eclipses that rose eclipsed, of the calculation of first contact times by the Late Babylonian astronomers.

or 10 degrees, but after this date it was the practice to quote to the nearest degree. Assuming this also to be true for predicted times (as would appear to be the case) we would expect the mean accuracy of prediction after 550 B.C. to be better than before this date for this reason alone. In fact, the mean accuracy of predicted eclipses after 550 B.C. is 0.95 hours (14 degrees). This is only marginally better, suggesting that there was no real gain in accuracy, despite the apparent increase in confidence in their own predictions on the part of the Babylonian astronomers.

Considering now the case of the near-miss predictions of penumbral eclipses, we again calculated the LT of prediction by determining the time of sunrise or sunset. Using modern ephemerides, we calculated the LT of the closest approach of the

Moon to the Earth's umbral shadow, as for a penumbral eclipse there are only virtual contacts. Table 4 gives the error in prediction for each of the near misses. In Figure 4 we plot the accuracy of the near-miss predictions. The mean error has been determined as -1.71 hours (-25 degrees), and the mean accuracy as 2.74 hours (41 degrees).

Finally, we look at the calculated times of first contact of eclipses that were observed to rise eclipsed. Table 5 lists these times and the error in first contact time, deduced as above in the case of umbral eclipses. The mean error in the calculation of first contact time is -0.1 hours (-2 degrees), and the mean accuracy is 0.50 hours (8 degrees). Figure 5 shows the accuracy of these first contact times. The fact that the accuracy of these calculations was significantly better than in the purely predicted cases leads us to conclude that these times were not obtained using the usual procedure for advance prediction.

Conclusion

In this paper we have analysed all of the predictions of lunar eclipses for which a time is recorded by the Late Babylonian astronomers. Considering first purely predicted eclipses, we divide the predictions into three categories: successful, near misses, and failures. In about 55% of cases the astronomers successfully predicted an umbral eclipse, 35% being near misses (penumbral), and the remaining 10% failures.

For the successful predictions, by proving that the time given in the prediction is first contact, we have been able to show that the typical accuracy is 1.12 hours, improving to 0.95 hours after 550 B.C. In the case of observed eclipses, at small time intervals the accuracy of the recorded times is much better than in the corresponding predicted cases, and is typically less than 0.5 hour for time-intervals up to 40 degrees.¹⁴ The greater accuracy of the observed times indicates that they were indeed measured and not simply based upon prediction.

The typical accuracy of the times of first contact for an observed eclipse that rose eclipsed was much better than for a prediction, being 0.50 hours. This suggests that these calculations of first contact time were not made using the usual cyclical procedure, but were retrospectively deduced from observations of the latter phases of the eclipse.

To conclude: the Late Babylonian astronomers had reasonable success in predicting lunar eclipses; certainly predictions of this standard would at least enable the astronomers to obtain a rough idea of when to look for an eclipse possibility.

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 In fact lunar eclipses in a Saros series recur every 18 years plus 10 or 11 days. The magnitudes of the eclipses increase from the start of the series to a peak before falling away at the end of the series. Typically a Saros series lasts for about 75 eclipses, about 25 of these being penumbral. For a more detailed discussion of Saros cycles see Bao-Lin Liu and A. D. Fiala, *Canon of lunar eclipses 1500 B.C. – A.D. 3000* (Richmond, Virginia, 1992), 8.

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