Physics 530-23
Assignment 1

1. Assume that $H^i{}_j$, $L_i{}^j{}_k$ and M_{ij} are tensors, and f, g are functions. Which of the following are tensor equations and why? (Recall that a tensor equation is on for which both sides of the equation transform in the same way under a coordinate transformation).
i)$Q_i{}^j = H^j{}_i$
ii)$R = H^i{}_i$
iii)$T^l_{ijk} = H^l{}_i M_{jk}$
iv)$T^l_{ijk} = H^l{}_i + M_{jk}$
v) $R^i = L_j{}^i{}_j$
vi) $S_i = L_i{}^j{}_j - L_j{}^j{}_i$

2. Given coordinates r, θ, what are the tangent vectors to the curves defined by the coordinate conditions

$$r(\lambda) = r_0 \tag{1}$$
$$\theta(\lambda) = \lambda \tag{2}$$

$$r(\lambda) = \lambda \tag{3}$$
$$\theta(\lambda) = 5\lambda \tag{4}$$

$$r(\lambda) = 10\lambda \tag{5}$$
$$\theta(\lambda) = 50\lambda \tag{6}$$

What is the cotangent vector of the following functions

$$f(r,\theta) = r^2 \tag{7}$$

$$f(r,\theta) = r^2 + \theta^2 \tag{8}$$

In each case find the lengths of these various vectors for each point at which they are defined if the metric is given by a)

$$ds^2 = dr^2 + d\theta^2 \tag{9}$$

and

$$ds^2 = dr^2 + r^2 d\theta^2 \tag{10}$$

3. Consider the two sets of coordinates x, y and r, θ where

$$r(x,y) = +\sqrt{x^2+y^2} \tag{11}$$
$$\tan(\theta) = \frac{y}{x} \tag{12}$$

What are x and y in terms of r and θ?

If we define x, y as x^1, x^2 and r, θ as $\tilde{x}^1, \tilde{x}^2$, what are the two Jacobian matrices

$$\partial_j \tilde{x}^i \text{ and } \partial_{\tilde{j}} x^i \tag{13}$$

If the metric for x, y is

$$ds^2 = dx^2 + dy^2 \tag{14}$$

What is the metric in terms of $r, \theta, dr, d\theta$?

4. Given that the metric for x^1, x^2, x^3 is

$$ds^2 = (dx^1)^2 + (dx^2)^2 + 4(dx^3)^2 \tag{15}$$

what are the components of the metric g_{ij}?

What are the components of g^{ij} and what is $\sqrt{g}$?

5. In cylindrical coordinates (r, θ, z), the metric is

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2 \tag{16}$$

Consider the vector potential $A_r = cos(\theta), \quad A_\theta = r\sin(\theta), \ A_z = 1$ Find the components of $B^i = \epsilon^{ijk}\partial_j A_k$

What are the components of B^i if we change the sign of A_θ in the above?