Physics 200-05

Practice 1

1). Show explicitly that if

$$\tilde{x} = cos(\theta)x + sin(\theta)y \quad (1)$$

$$\tilde{y} = cos(\theta)y - sin(\theta)x \quad (2)$$

then the distance from the origin in the $\tilde{x},\ \tilde{y}$ system is the same as in the $x,\ y$ system.

2) [Hard] Show that if $\tilde{x} = X(x,y),\ \ \tilde{y} = Y(x,y)$, then the requirement that the distance between any two nearby points x_1, y_1 and x_2, y_2 be the same as between $\tilde{x}_1, \tilde{y}_1$ and $\tilde{x}_2, \tilde{y}_2$, for all x_1, y_1 and nearby x_2, y_2 is that

$$\left(\frac{\partial X}{\partial x}\right)^2 + \left(\frac{\partial Y}{\partial x}\right)^2 = \left(\frac{\partial X}{\partial y}\right)^2 + \left(\frac{\partial Y}{\partial y}\right)^2 = 1 \quad (3)$$

$$\left(\frac{\partial X}{\partial x}\right)\left(\frac{\partial X}{\partial y}\right) = -\left(\frac{\partial Y}{\partial y}\right)\left(\frac{\partial Y}{\partial x}\right) \quad (4)$$

and that the second derivatives of X and Y are all zero. (Expand the expression for the distance in the $\tilde{x}, \tilde{y}$ coordinates in a taylor series in terms of the the $x_2 - x_1$ and $y_2 - y_1$).

This means that the only transformations must be of the form

$$X(x,y) = cos(\theta)x + sin(\theta)y + c_x \quad (5)$$

$$Y(x,y) = cos(\theta)y - sin(\theta)x + c_y \quad (6)$$

where the c_x and c_y are constants and

$$cos(\theta) = \frac{\partial X}{\partial x} = \frac{\partial Y}{\partial y}. \quad (7)$$

Ie, in two dimensions, the only transformations of the coordinates which keeps all distances the same are rotations and translations.

3) (Abberation) Rain is falling vertically and hits the ground with speed c. A bicyclist is travelling through the rain with velocity $c/2$. At what angle (from the vertical) does the cyclist feel the rain as hitting him?

4)Assume that the aether is completely dragged by light. Thus the velocity of light in water flowing with the light is $\frac{c}{n} + v$ while that for light in water flowing against the light is $\frac{c}{n} - v$. What would be the difference in the time (to lowest order in v) it takes light to traverse two meters of flowing water, if the water is flowing at 10 m/sec. (recall that the velocity of light, $c = 3 \cdot 10^8 m/sec$ and the index of refraction of water is 1.3. If the frequency of light used is $2 \cdot 10^{15} Hz$ what is this difference in time as a fraction of the period of the light.

Fresnels theory says that the drag is not v, but rather is (to lowest order in v) $v(1 - \frac{1}{n^2})$. How much of a difference would this make in the above exeriment?(See text book).