5 The Formation of Quantum Mechanics

5.1 The Rise of Matrix Mechanics

In spite of its high-sounding name and its successful solutions of numerous problems in atomic physics, quantum theory, and especially the quantum theory of polyelectronic systems, prior to 1925, was, from the methodological point of view, a lamentable hodgepodge of hypotheses, principles, theorems, and computational recipes rather than a logical consistent theory. Every single quantum-theoretic problem had to be solved first in terms of classical physics; its classical solution had then to pass through the mysterious sieve of the quantum conditions or, as it happened in the majority of cases, the classical solution had to be translated into the language of quanta in conformance with the correspondence principle. Usually, the process of finding "the correct translation" was a matter of skillful guessing and intuition rather than of deductive and systematic reasoning. In fact, quantum theory became the subject of a special craftsmanship or even artistic technique which was cultivated to the highest possible degree of perfection in Göttingen and in Copenhagen. In short, quantum theory still lacked two essential characteristics of a full-fledged scientific theory, conceptual autonomy and logical consistency.

The core of the difficulties was, of course, the fact that according to classical physics, which served as the point of departure for quantum-theoretic calculations as long as atomic systems were described in classical terms, the optical frequencies of spectral lines should coincide with the Fourier orbital frequencies of the system's motions, a result not borne out by experiment. The discrepancy was smoothed over by Bohr's heuristically invaluable principle of correspondence. For it made it possible to retain

the description of motion in terms of classical kinematics and dynamics but allowed at the same time a certain tailoring of the results so as to fit it to observational data. It was Heisenberg who recognized that this approach is only one alternative. The other alternative, which he chose in his historic paper "On a quantum theoretical interpretation of kinematical and mechanical relations"[1] and which led to the development of matrix mechanics, the earliest formulation of modern quantum mechanics, abandoned Bohr's description of motion in terms of classical physics altogether and replaced it by a description in terms of what Heisenberg regarded as observable magnitudes.

As the following analysis attempts to show, Heisenberg's crucial intervention, if examined with respect to its epistemological tenets, seems to have been made under the influence of those intellectual currents which have been mentioned in our discussion on the philosophical background of nonclassical conceptions. Bohr, it will be recalled,[2] gave a series of lectures in 1922 at the University of Göttingen, whose influence upon Pauli has already been pointed out. These lectures were also attended by Heisenberg. In one of these lectures Bohr spoke on the quadratic Stark effect and expressed his confidence that the theory in its present formulation is on the right track in spite of the as yet unclarified contradictions and the impossibility of calculating exactly the intensities of spectral lines. In particular, Bohr declared, the great success in explaining the linear Stark effect on the basis of the quantum conditions made him feel sure that the proposed interpretation of the quadratic effect must also be correct. Heisenberg, who had already begun his study of the problem of dispersion, differed with Bohr on this issue. Recognizing that the quadratic Stark effect may be considered as a limiting case of dispersion for incident radiation of infinitely low frequency and realizing that a quantum-theoretic treatment of dispersion cannot be worked out along the lines proposed, Heisenberg criticized Bohr's statement. Although Bohr, as Heisenberg recalls,[3] discussed with him privately, after the lecture, this problem and related questions for over three hours, Heisenberg did not retract his challenge. On the contrary, Bohr's frequent remarks that "the experimental situation has to be covered by means of concepts which fit"—in consonance with the Kierkegaard-Høffding insistence that every field of experience required its own conceptions and principles and, interestingly, also in consonance with the philosophies of Comte and Boutroux—seem to have impressed the young Heisenberg strongly and encouraged him in his search

[1] W. Heisenberg, "Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen," *Zeitschrift für Physik 33*, 879–893 (1925); *Dokumente der Naturwissenschaft*, vol. 2 (Ernst Battenberg Verlag, Stuttgart, 1962), pp. 31–45.

[2] In this context see REF. 107 OF CHAP. 1.

[3] *Archive for the History of Quantum Physics*, Interview with W. Heisenberg on Nov. 30, 1962.

for an alternative approach. Heisenberg admitted repeatedly that it had been Bohr's influence and thus, ultimately, the philosophical presumptions underlying Bohr's conception of physics which had suggested to him the idea that the prevailing conceptual apparatus was not a categorical necessity or an indubitable catechism.

Thus if Heisenberg's conception of the very possibility of rejecting the description of atomic systems in terms of classical physics may be traced back, via Bohr, to one of the schools of philosophical thought referred to previously, his choice of the particular nature of the new concepts by which he replaced the classical ones goes back to the other philosophical trend mentioned, the positivism or logical empiricism of the early twenties. Interested in philosophy while still a student at the classical "gymnasium"—his first acquaintance with atomic theory was through Plato's *Timaeus*—he was strongly impressed, first by Kant's *Critique of Pure Reason* and later particularly by Wittgenstein's writings. But what most appealed to his mind was Einstein's treatment of the concept of time, his replacement of Lorentz's "local" or "mathematical" time in the so-called Lorentz transformations[4] by the operationally defined and, in this sense observable, relativistic time. Einstein's rejection of the Newtonian, operationally undefinable, conception of simultaneity of spatially separated events and his relativistic reinterpretation of the Lorentz-FitzGerald contraction, previously regarded as an effect unobservable in principle, undoubtedly made a strong impression on Heisenberg. It is known that both in Munich, where Heisenberg had written his thesis under the guidance of Sommerfeld, and in Göttingen, where he was working under Born and where Minkowski lectured, relativity was studied with great fervor.

The preceding remarks will make it clear why Heisenberg insisted on using observable magnitudes as terms for the description of atomic states. From the modern sophisticated point of view a distinction between observable and theoretically inferred magnitudes poses, of course, a highly complicated problem. In fact, when in 1926 Heisenberg confided to Einstein that "the idea of observable quantities was actually taken from his relativity,"[5] Einstein already pointed out that it is the theory which ultimately decides what can be observed and what cannot. One may admit, however, that such a distinction—even if it is, prior to the establishment of a theory, unwarranted—can be adopted as merely a heuristic principle.

Although the explicit formulation of these deliberations was, of course,

[4] The transformation equations as such had already been employed in 1887 by W. Voigt in his study of the Doppler effect for vibrations in an elastic and incompressible medium. See W. Voigt, "Ueber das Dopplersche Princip," *Göttinger Nachrichten 1887*, pp. 41–51. The term "Lorentz transformation" was introduced by H. Poincaré in his paper "Sur la dynamique de l'électron," *Comptes Rendus 140*, 1504–1508 (1905).

[5] *Archive for the History of Quantum Physics*, Interview with W. Heisenberg on Feb. 15, 1963.

of a later date, it was precisely this heuristic approach which Heisenberg adopted, as the following discussion tries to show. Heisenberg rejected the classical notions of position and velocity or momentum of electrons in atoms not only because they were unobservable in the sense that, so far, nobody had measured them directly. For it may always be argued, as Heisenberg explicitly admitted, that future progress in experimental techniques would eventually make it possible to measure these quantities. "This hope," continued Heisenberg—and this is the essential point—"could be regarded as justified if the formal theory [in terms of which these quantities are calculated] can consistently be applied to a distinctly defined domain of quantum-theoretic problems. Experience, however, shows that . . . the reaction of atoms to periodically varying fields cannot be described by these rules nor is their application to polyelectronic systems feasible."[6] Thus, Heisenberg's rejection of these quantities as unobservable was based on two empirical facts, the experimental impossibility of directly measuring them *and* the practical failure of a theory which assumed them to be observable. He replaced these quantities of classical kinematics by the optical quantities of frequency and intensity, or rather dipole amplitude,[7] and investigated whether a theory, assuming these as observable, can be worked out consistently. Basically, Heisenberg's attitude, in this respect, resembled that of Einstein, for whom the concept of Newtonian time had lost its physical significance not only, as he showed in his analysis of the simultaneity of spatially separated events, because of its insusceptibility to operational determination but also because classical physics which assumed this concept as observable conflicted with experience.

A second fundamental innovation in Heisenberg's approach was the way he employed Bohr's correspondence principle. As mentioned before, the translation of classical formulas into the language of quantum theory usually required a great deal of guessing and had only one guideline, the correspondence principle. It had to be used in a separate way for almost every problem, the particular mode of its application depending on the specific data of the problem. While its versatility and fertility made it attractive to the more synthetically minded physicists, such as Bohr, its flexibility and lack of rigidity made it repugnant to the more analytically oriented theoreticians, such as Sommerfeld. Heisenberg, influenced by both Sommerfeld and Bohr, considered now the possibility of "guessing"[8]—in accordance with the correspondence principle—not the solution of a par-

[6] REF. 1, p. 879.

[7] His study of the dispersion problem suggested that not only the squares of the magnitudes of these amplitudes (the intensities) but also their phases are experimentally ascertainable.

[8] "Also war es vielleicht auch möglich, einfach durch geschicktes Erraten eines Tages den Übergang zum vollständigen mathematischen Schema der Quantenmechanik zu vollziehen." W. Heisenberg, "Erinnerungen an die Zeit der Entwicklung der Quantenmechanik," in *Theoretical Physics in the Twentieth Century*, edited by M. Fierz and V. F. Weisskopf (Interscience, New York, 1960), p. 42.