Physics 200-05
Assignment 2

1. i) Show that

$$\sinh(\theta)\cosh(\theta') + \cosh(\theta)\sinh(\theta') = \sinh(\theta+\theta') \tag{1}$$
$$\cosh(\theta)\cosh(\theta') + \sinh(\theta)\sinh(\theta') = \cosh(\theta+\theta') \tag{2}$$

Note the similarities **and** differences with the trigonometric formulas you are (I hope) more familiar with.

ii) Using the above formula show that two successive Lorentz transformations both along the x direction are such that if the velocity of transformation from the first to the second frame is v_1 and of the second to the third frame is v_2 then the velocity from the first to third frame is

$$\frac{v_f}{c} = \frac{\frac{v_1}{c} + \frac{v_2}{c}}{1 + \frac{v_1 v_2}{c^2}} \tag{3}$$

(Use the fact that $\tanh(\theta) = \frac{v}{c}$). Ie, while the rapidities (the "angle" in the hyperbolic function representation of the Lorentz transformations) of successive Lorentz transformations add, the velocities do not.

2.) Show the consistancy of the special relativistic formulas. Consider the following synchronization of clocks. Alice and her friend Amy get together at the origin and sychronize their clocks to each other, ensuring that both show exactly the same time and run at the same rate. Now Amy very slowly (with a velocity δv approaching zero) moves away from Alice to a location X along Alice's x axis. Show that in Alice's frame, the time on Amy's clock at X will be synchronized with her clock. (Show that in the limit as δv goes to zero, the time difference between Amy's time to get to the location X and Alice's time for Amy to get to X are the same.) Now let us look at this process from Bob's point of view, who is moving with velocity v with respect to Alice. Show that Bob will calculate the difference between Amy's time to get to X and Alice's time is $\frac{vX}{c^2\sqrt{1-\frac{v^2}{c^2}}}$.

Hint, use the expression for the rate of Alice's and Amy's clocks according to Bob (time dilation) and look at the difference to lowest order in δv. Find the time it takes Amy to get to the point X and express the time difference in terms of X.

3. Let us assume that we measure time in units (light meters) such that the velocity of light is 1. Consider the following pairs of points. Are they timelike, spacelike or null separated and what is the squared distance between them?

i) $t=0;\ x=1;\ y=1;\ z=0$ and $t=1;\ x=1;\ y=0;\ z=0$

ii) $t=2;\ x=3;\ y=1\ ; z=0$ and $t=3;\ x=2;\ y=3;\ z=0$

iii) $t=3;\ x=1;\ y=1;\ z=0;$ and $t=5;\ x=1;\ y=0;\ z=0$

In each case, what velocity and in what direction, would be needed to make the time separated points occur at the same point in space, or the spacelike separated points occur at the same time?

4) i) Two particles are travelling at 4/5 the velocity of light and are 5 cm apart. How far apart are they in the frame in which they are at rest?

ii) Muons are created in the upper atmosphere of the earth (say 10km high). How fast would they have to be travelling so that half of them reached the surface of the earth?

5)4) Peter Spacerider has heard about Relativity and heard that from the point of view of a rapidly travelling observer, his own spaceship is really short. He passes a spaceship identical to his own travelling in the opposite directioni at almost the velocity of light. Just as the nose of his spaceship is at the tail of the other spaceship he presses the button in the nose to fire the laser canon in the tail of his own spaceship at the other spaceship. His collegue Johnny says "You are an idiot. It is the other spaceship that is really short since it is travelling with respect to us. Your shot has missed." Who is right? Why? What is wrong with the other's argument. (Note, you can assume that the distance between the spaceships passing each other is much less than the length of the spaceships.)

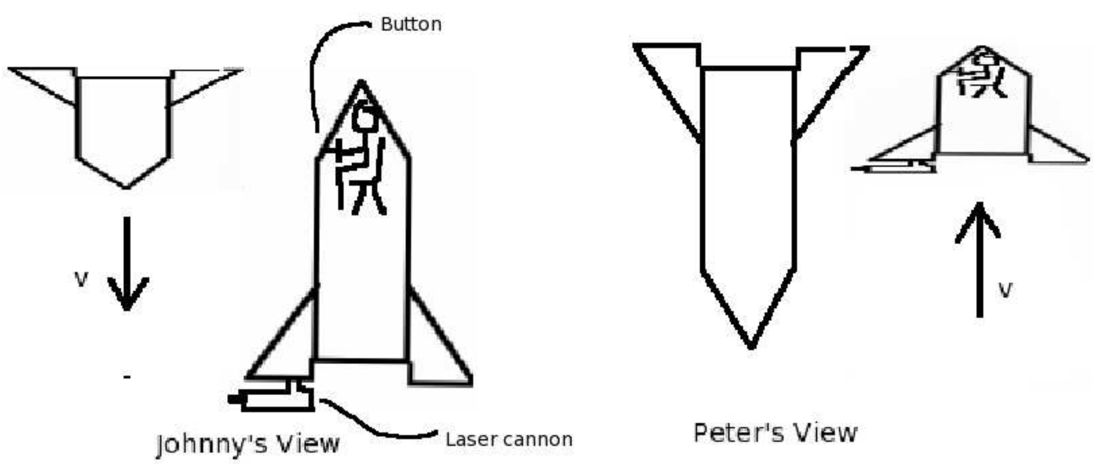

Figure 1: For Problem 5. The two views of the spaceships passing each other. Peter is in the nose of the right spaceship where he presses the button to fire the laser canon in the tail of his spaceship.